‘Mathematics is the key and door to the sciences.’

– Galileo Galilei

MATHEMATICS
LESSON PLAN
GRADE 10 TERM 4

A MESSAGE FROM NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers

This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE). We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

WHAT IS NECT?

In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education.

The NECT has successfully brought together groups of people interested in education so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

WHAT ARE THE LEARNING PROGRAMMES?

One of the programmes that the NECT implements on behalf of the DBE is the 'District Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). Curriculum learning programmes were developed for Maths, Science and Language teachers in FSS who received training and support on their implementation. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers.

The FSS helped the DBE trial the NECT learning programmes so that they could be improved and used by many more teachers. NECT has already begun this scale-up process in its Universalisation Programme and in its Provincialisation Programme.

Everyone using the learning programmes comes from one of these groups; but you are now brought together in the spirit of collaboration that defines the manner in which the NECT works. Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

www.nect.org.za

CONTENTS

PROGRAMME ORIENTATION

Welcome!

The NECT FET Mathematics Learning Programme is designed to support teachers by providing:

- Lesson Plans
- Trackers
- Resource Packs
- Assessments and Memoranda
- Posters.

This Mathematics Learning Programme takes care of most of the planning required to teach FET Mathematics. However, it is important to remember that although the planning has been done for you, preparation is key to successful teaching. Ensure that you put aside adequate time to properly prepare to teach each topic.

Also remember that the most important part of preparation is ensuring that you develop your own deep conceptual understanding of the topic. Do this by:

- working through the lesson plans for the topic
- watching the recommended video clips at the end of the topic
- completing all the worked examples in the lesson plans
- completing all activities and exercises in the textbook

If, after this, a concept is still not clear to you, read through the section in the textbook or related teacher's guide, or ask a colleague for assistance. You may also wish to search for additional teaching videos and materials online.

Orientate yourself to this Learning Programme by looking at each component, and by taking note of the points that follow.

TERM 4 TEACHING PROGRAMME

1. In line with CAPS, the following teaching programme has been planned for FET Mathematics for Term 4:

Grade 10		Grade 11		Grade 12	
Topic	**No. of weeks**	**Topic**	**No. of weeks**	**Topic**	**No. of weeks**
Probability	2	Statistics	3	Revision	3
Revision	4	Revision	3		

2. Term 4 lesson plans and assessments are provided for six weeks for Grades 10 and 11.
3. Term 4 assessments are provided for six weeks for Grade 12
4. Each week includes 4,5 hours of teaching time, as per CAPS.
5. You may need to adjust the lesson breakdown to fit in with your school's timetable.

LESSON PLAN STRUCTURE

The Lesson Plan for each term is divided into topics. Each topic is presented in exactly the same way:

TOPIC OVERVIEW

1. Each topic begins with a brief **Topic Overview**. The topic overview locates the topic within the term, and gives a clear idea of the time that should be spent on the topic. It also indicates the percentage value of this topic in the final examination, and gives an overview of the important skills and content that will be covered.
2. The **Lesson Breakdown Table** is essentially the teaching plan for the topic. This table lists the title of each lesson in the topic, as well as a suggested time allocation.
   For example:

	Lesson title	Suggested time (hours)
1	Revision	2,5
2	Venn diagrams	2,5
3	Inclusive and mutually exclusive events; Complementary and Exhaustive events	1,5
4	Revision and Consolidation	1,5

4. The **Sequential Table** shows the prior knowledge required for this topic, the current knowledge and skills to be covered, and how this topic will be built on in future years.
   - Use this table to think about the topic conceptually:
     - Looking back, what conceptual understanding should learners have already mastered?
     - Looking forward, what further conceptual understanding must you develop in learners, in order for them to move on successfully?
   - If learners are not equipped with the knowledge and skills required for you to continue teaching, try to ensure that they have some understanding of the key concepts before moving on.
   - In some topics, a revision lesson has been provided.

5. The **NCS Diagnostic Reports.** This section is potentially very useful. It lists common problems and misconceptions that are evident in learners' NSC examination scripts. The Lesson Plans aim to address these problem areas, but it is also a good idea for you to keep these in mind as you teach a topic.

6. The **Assessment of the Topic** section outlines the formal assessment requirements as prescribed by CAPS for Term 4.

Grade	Assessment requirements for Term 4 (as prescribed in CAPS)
10	Test, Examination Paper I and Paper II
11	Test, Examination Paper I and Paper II
12	Examination Paper I and Paper II

7. The glossary of **Mathematical Vocabulary** provides an explanation of each word or phrase relevant to the topic. In some cases, an explanatory sketch is also provided. It is a good idea to display these words and their definitions or sketches somewhere in the classroom for the duration of the topic. It is also a good idea to encourage learners to copy down this table in their free time, or alternately, to photocopy the Mathematical Vocabulary for learners at the start of the topic. You should explicitly teach the words and their meanings as and when you encounter these words in the topic.

INDIVIDUAL LESSONS

1. Following the **Topic Overview**, you will find the **Individual Lessons.** Each lesson is structured in exactly the same way. The routine within the individual lessons helps to improve time on task, and therefore, curriculum coverage.

2. In addition to the lesson title and time allocation, each lesson plan includes the following:

A. ***Policy and Outcomes***. This provides the CAPS reference, and an overview of the objectives that will be covered in the lesson.

B. ***Classroom Management***. This provides guidance and support as you plan and prepare for the lesson.

- Make sure that you are ready to begin your lesson, have all your resources ready (including resources from the Resource Pack), have notes written up on the chalkboard, and are fully prepared to begin.
- Classroom management also suggests that you plan which textbook activities and exercises will be done at which point in the lesson, and that you work through all exercises prior to the lesson.
- In some cases, classroom management will also require you to photocopy an item for learners prior to the lesson, or to ensure that you have manipulatives such as boxes and tins available.
  The Learner Practice Table. This lists the relevant practice exercises that are available in each of the approved textbooks.
- It is important to note that the textbooks deal with topics in different ways, and therefore provide a range of learner activities and exercises. Because of this, you will need to plan when you will get learners to do the textbook activities and exercises.
- If you feel that the textbook used by your learners does not provide sufficient practice activities and exercises, you may need to consult other textbooks or references, including on0line references.
- The *Siyavula* Open Source Mathematics textbooks are offered to anyone wishing to learn mathematics and can be accessed on the following website: https://www.everythingmaths.co.za/read

C. ***Conceptual Development****:*
This section provides support for the actual teaching stages of the lesson.

Introduction: This gives a brief overview of the lesson and how to approach it. Wherever possible, make links to prior knowledge and to everyday contexts.

Direct Instruction: Direct instruction forms the bulk of the lesson. This section describes the teaching steps that should be followed to ensure that learners develop conceptual understanding. It is important to note the following:

- Grey blocks talk directly to the teacher. These blocks include teaching tips or suggestions.
- Teaching is often done by working through an example on the chalkboard. These worked examples are always presented in a table. This table may include grey cells that are teaching notes. The teaching notes help the teacher to explain and demonstrate the working process to learners.

- As you work through the direct instruction section, and as you complete worked examples on the chalkboard, ensure that learners copy down:
  - formulae, reference notes and explanations
  - the worked examples, together with the learner's own annotations.
- These notes then become a reference for learners when completing examples on their own, or when preparing for examinations.
- At relevant points during the lesson, ensure that learners do some of the Learner Practice activities as outlined at the beginning of each lesson plan. Also, give learners additional practice exercises and questions from past papers as homework. Ensure that learners are fully aware of your expectations in this respect.

D. ***Additional Activities / Reading***. This section provides you with web links related to the topic. Get into the habit of visiting these links as part of your lesson preparation. As teacher, it is always a good idea to be more informed than your learners. If possible, organise for learners to view video clips that you find particularly useful.

THE REVISION PROGRAMME

The teaching programme for FET mathematics Term 4 differs from the teaching programmes for Terms 1-3. There is only one topic with new content in Term 4 for Grades 10 and 11; and no new content in Term 4 for Grade 12. Most of the contact time in Term 4 is allocated to consolidation, revision and preparation for the end of year examinations. The Revision Programme for each grade are designed to support you and the learners so as to ensure that revision time is effectively and productively used.

THE STRUCTURE OF THE REVISION PROGRAMME:

- Summary notes for the topics assessed in Paper I and Paper II. These notes are provided in the Resource Pack. If possible, the summary notes should be photocopied for learners. Alternatively, you could provide learners with an electronic copy of the summary notes; or learners can copy down the summary notes. Encourage learners to add their own notes to the summary notes you have given them.
- Fully worked past paper.
- Past papers and memoranda. The past papers and memoranda are provided in the Resource Pack. If possible, the past papers, exemplars and memoranda should be photocopied for learners. Alternatively, you could provide learners with an electronic copy of the examinations, exemplars and memoranda; or learners can share copies. The links to these resources are provided in the Lesson Plan.

Working through past papers and exemplars has been shown to be an excellent learner-centred approach to revision. For this reason, we urge you to do everything possible to ensure that learners have access to these materials.

TRACKER

1. A Tracker is provided for Grades 10 and 11 for Term 4. The Trackers are CAPS compliant in terms of content and time.
2. You can use the Tracker to document your progress. This helps you to monitor your pacing and curriculum coverage. If you fall behind, make a plan to catch up.
3. Fill in the Tracker on a daily or weekly basis.
4. At the end of each week, try to reflect on your teaching progress. This can be done with the HoD, with a subject head, with a colleague, or on your own. Make meaningful notes about what went well and what didn't. Use the reflection section to reflect on your teaching, the learners' learning and to note anything you would do differently next time.
   These notes can become an important part of your preparation in the following year.

RESOURCE PACK, ASSESSMENT AND POSTERS

1. A Resource Pack with printable resources has been provided for each term.
2. These resources are referenced in the lesson plans, in the Classroom Management section.
3. Two posters have been provided as part of the FET Mathematics Learning Programme for Term 4.
4. Ensure that the posters are displayed in the classroom.
5. Try to ensure that the posters are durable and long-lasting by laminating it, or by covering it in contract adhesive.
6. Note that you will only be given these resources once. It is important for you to manage and store these resources properly. You can do this by:
   - Writing your school's name on all resources
   - Sticking resource pages onto cardboard or paper
   - Laminating all resources, or covering them in contact paper
   - Filing the resource papers in plastic sleeves once you have completed a topic.
7. Add other resources to your resource file as you go along.
8. Note that these resources remain the property of the school to which they were issued.

ASSESSMENT AND MEMORANDUM

In the Resource Pack you are provided with assessment exemplars and memoranda as per CAPS requirements for the term. For Term 4, the Resource Pack contains one test and memorandum for Grades 10 and 11. In addition, past papers, exemplars and memoranda are provided for Grades 10, 11 and 12.

CONCLUSION

Teacher support and development is a complex process. For successful Mathematics teachers, certain aspects of this Learning Programme may strengthen your teaching approach. For emerging Mathematics teachers, we hope that this Learning Programme offers you meaningful support as you develop improved structure and routine in your classroom, develop deeper conceptual understanding in your learners and increase curriculum coverage.

Term 4, Topic 1: Probability

TOPIC OVERVIEW

TOPIC OVERVIEW

A

- This topic is the only topic in Term 4.
- This topic runs for two weeks (9 hours).
- It is presented over four lessons.
- The lessons have been divided according to sub-topics, not according to one school lesson. An approximate time has been allocated to each lesson (which will total 9 hours). For example, one lesson in this topic could take two school lessons. Plan according to your school's timetable.
- Probability counts 15% of the final Paper 1 examination.
- Traditionally, probability either gets taught extremely well or almost not at all. If you feel that your knowledge of this topic is not as good as it could be, watch videos and read up on the concepts required.
- Watch the following video for some inspiration: https://www.ted.com/talks/arthur_benjamin_s_formula_for_changing_math_education (Arthur Benjamin is a professor of mathematics in the United States. He discusses the fact that most topics in school mathematics lead to being able to learn calculus. However, he believes that statistics and probability are more important and that calculus can always be studied in more detail by students of mathematics who go on to study mathematics at tertiary level).

Breakdown of topic into 4 lessons:

	Lesson title	Suggested time (hours)		Lesson title	Suggested time (hours)
1	Revision	2,5	3	Inclusive and mutually exclusive events; Complementary and Exhaustive events	2,5
2	Venn diagrams	2,5	4	Revision and Consolidation	1,5

B SEQUENTIAL TABLE

GRADE 8 & 9	GRADE 10	GRADE 11 & 12
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
• Determine probabilities for compound events using two-way tables and tree diagrams • Determine the probabilities for outcomes of events and predict their relative frequency in simple experiments • Compare relative frequency with probability Only situations with equally probable outcomes are considered.	• Compare relative frequency and theoretical probability • Use of Venn diagrams to solve problems • Mutually exclusive events • Complementary events • Derive and using the identity: $P(A \text{ or } B)$ $= P(A) + P(B) – P(A \text{ and } B)$	• Revision of addition rule for mutually exclusive events • Dependent and independent events • Use of Venn diagrams, tree diagrams and contingency tables to solve problems • The fundamental counting principle • Probability problems using the fundamental counting principle.

C WHAT THE NSC DIAGNOSTIC REPORTS TELL US

According to **NSC Diagnostic Reports** there are several issues pertaining to Probability.

These include:

- Confusion between n(A) and P(A)
- Not reading from a table correctly
- Incorrect use of notation (for example, P(0,2)).

It is important for you to keep these issues in mind when teaching this section.

Explain the concepts in depth and always use the correct notation.

D ASSESSMENT OF THE TOPIC

- CAPS formal assessment requirements for Term 4:
  - Test
  - Examination (Paper 1 & Paper 2)
- A test, with memorandum, is provided in the Resource Pack. The test is aligned to CAPS in every respect, including the four cognitive levels as required by CAPS (page 53). The test contains questions on Probability as well as Algebra (for revision).
- The questions usually take the form of problems relating to the probability of an event and can include Venn diagrams, tree diagrams or tables.
- Monitor each learner's progress to assess (informally) their grasp of the concepts. This information can form the basis of feedback to the learners and will provide you valuable information regarding support and interventions required.

E MATHEMATICAL VOCABULARY

Be sure to teach the following vocabulary at the appropriate place in the topic:

Term	Explanation
probability	The likelihood or chance of something happening. A probability answer is ALWAYS in the range: $0 \leq x \leq 1$
trial/experiment	The process of trying something out to find the chance (probability) of an event occurring. For example: Tossing a coin 100 times
outcome	A possible result from an experiment. For example: 'tails' is one of two possible outcomes when tossing a coin
sample space	The set of all possible outcomes of an experiment
experimental probability	The result of doing an experiment to find the chances of an event occurring For example: An experiment was conducted to see how many tails appeared when a coin was tossed 100 times. The result was $\frac{47}{100}$
relative frequency	The outcome of an experiment In the experimental probability example, $P(A)=\frac{n(A)}{n(S)}$ is the relative frequency
theoretical probability	The probability of an event happening using knowledge of numbers. The theoretical calculation
tree diagram	Method used for counting the number of possible outcomes of an event. The last column of the tree diagram shows all the possible outcomes

contingency table	Table showing the distribution of one variable in rows and another in columns, used to study the correlation between the two variables
Venn diagram	Useful way to represent mathematical or logical sets of information. In a Venn diagram, the position and overlapping of circles are used to indicate the relationships between different sets of information
union	The set of all outcomes that occur in at least one of the events. Key word: or
intersection	The set of outcomes that occur in all the events Key word: and
event	Set of outcomes of an experiment (a subset of the sample space) to which a probability is assigned
mutually exclusive events	Events with no outcomes in common (no intersection) Events that cannot happen at the same time.
complementary events	Mutually exclusive events that contain all the outcomes between them. They are the only two possible outcomes
exhaustive events	The union of two events is equal to the entire sample space – at least one of the events must occur.
independent events	Two events where the outcome of one event does not affect the outcome of the other
dependent events	The outcome of one event affects the outcome of the next event
element	Member of a set

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Term 4, Topic 1, Lesson 1

REVISION

Suggested lesson duration: 2,5 hours

POLICY AND OUTCOMES

A

CAPS Page Number	29
Lesson Objectives By the end of the lesson, learners will have revised: ● relative frequency vs theoretical probability ● tree diagrams ● Venn diagrams ● two-way tables.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. You will need Resource 1 from the Resource Pack. Distribute this investigation to learners the day before you plan on starting this section.
4. Write the lesson heading on the board before learners arrive.
5. Write work on the chalkboard before the learners arrive.
6. For this lesson, be prepared for the activity in point 10. Ensure you have the items and a bag available. You will need three cards with the word BLUE on each card and four cards with the word RED on each card.
7. If there isn't a revision exercise in the textbook that you use, either use the revision exercise at the end of a Grade 9 textbook or items from a Grade 9 test on Probability.

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LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
1	295	1	291			17.1 17.4	378 389	14.1 14.2	474 477

C

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. It is important that learners have the required knowledge from previous grades before doing Grade 10 work. Do not move on to Grade 10 work until you are sure that the learners are confident with the basics. Address knowledge gaps where necessary.
2. There is plenty of time available for this lesson. Give learners time to work in class where you can be available to assist and support them where necessary.

DIRECT INSTRUCTION

1. Start the lesson by asking: W*hat can you tell me about the answer to any probability question?*
   (The answer can never be less than zero or greater than 1).
2. Ask: *What do you remember from Probability in Grade 8 or Grade 9?*
3. Listen carefully to learners' responses. Do not dismiss any ideas or examples.
   Write the key points mentioned on the chalkboard. If any of the following are not mentioned, ask directed questions to remind learners what they should know.
   - relative frequency vs theoretical probability
   - tree diagrams
   - Venn diagrams
   - two-way tables.
4. Ask learners to refer to the investigation they completed at home.

5. Ask: *Did you all get the same results when tossing the coin 30 times?*
   (No. This is highly unlikely).

6. Allow learners a minute or two to compare their results with a partner.

7. If it has not yet been mentioned, ask:
   What is the difference between relative frequency and theoretical probability? Give me an example.

There may be different responses to these questions. Agree or correct learners where necessary. Allow a few learners to give an example.

8. Confirm the following and ask learners to write the definitions in their books under the heading, Probability.
   Theoretical probability: The probability of an event happening using knowledge of numbers.
   The formula to find the probability of event A:
   $P(A) = \frac{n(A)}{n(S)}$
   Another way of writing it:
   $\mathrm{P(event)} = \frac{\text{the number of ways the event could happen}}{\text{the total number of possible equally likely outcomes}}$
   $n(A)$ is the number of possibilities in event A.
   $n(S)$ is the number of possibilities in the entire sample space.

   For example, the probability of throwing a 2 when throwing a six-sided die is $\frac{1}{6}$.

   The 1 is because there is only one two on the die and the 6 is the sample space – all the possibilities that can occur when throwing a die.

   Relative frequency: The outcome of an experiment. Refer learners to their investigation.

9. Summarise the idea for learners: Theoretical probability is what we EXPECT to occur whereas the relative frequency is what ACTUALLY occurs if the experiment is carried out.

10. Use the seven cards that you prepared. Show them to learners saying: *Look at the three blue cards and the four red cards I have. I am putting all seven cards inside the bag.*

11. Ask: *What is the probability that a red card is chosen?*
    $\left(\frac{4}{7}\right)$

    Why?
    (There are four red cards out of a total of seven in the bag).
    Ask: *What is the probability that a blue card is chosen?*
    $\left(\frac{3}{7}\right)$

    Why?
    (There are three blue cards out of a total of seven in the bag).

12. After you have given the bag a shake, ask a learner to reach into the bag and take out one card (WITHOUT looking into the bag).

    Ask: *If I **don't** put this card back in the bag, will it affect the probabilities next time someone draws a card?*
    (Yes)

    Ask: *How*?
    (The total number of cards is now six and no longer seven. One set of cards remains the same amount and the other set goes down by one).

> The second part of the answer depends on what colour card the learner picked:
>
> - If a red card was picked, there will now be three red cards and still three blue cards.
> - If a blue card was picked, there will now be two blue cards and still four red cards.
>
> If a red card was picked, use this opportunity to discuss the fact that the next time someone picks a card there will be 50/50 (equal) chance of getting a red or a blue card. This is because there is now the same number of red and blue cards (three of each).
> If a blue card was chosen, use this opportunity to discuss the fact that there is a much better chance of getting a red card next time someone picks a card.

13. Say: *Note the way in which replacing the card, or not replacing the card, affects how we calculate the probability for the next event.*

14. Tell learners that the key words here are *dependent events* and *independent events.*

15. Learners should write these definitions in their exercise books:
    Dependent events: The outcome of one event affects the outcome of the next event.
    Independent events: Two events where the outcome of one event does not affect the outcome of the other.

16. Ask: *What is the probability that a yellow card will be drawn from the bag?*
    (Zero – there are no yellow cards, so it is impossible).

17. Ask: *What is the probability that a card will be drawn from the bag?*
    (One – every item in the bag is a card, so it is certain that a card will be drawn).

18. Ask learners to write this summary in their exercise books:
    If something has a probability of 1, it is <u>CERTAIN</u> to happen
    If something has a probability of 0, it is <u>IMPOSSIBLE</u>

19. A key concept towards understanding probability is that the sum of all the possible outcomes of an event will be 1. Use the bag of cards to explain this concept further:
    There was a $\frac{4}{7}$ chance of drawing a red card and a $\frac{3}{7}$ chance of drawing a blue card and $\frac{4}{7}+\frac{3}{7}=1$ because these two events (drawing a red card and drawing a blue card) cover all the possibilities.

20. Use this example to assist learners in understanding the idea of the probability of an event <u>not</u> happening:

    You have 5 T-shirts. One of them is red. You pull a T-shirt from the drawer WITHOUT looking.
    Ask: *What is the probability of pulling out a red T-shirt?* $\left(\frac{1}{5}\right)$
    Ask: *What is the probability of not pulling out the red T-shirt?* $\left(\frac{4}{5}\right)$

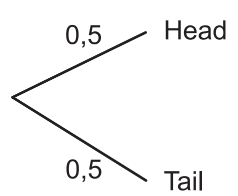
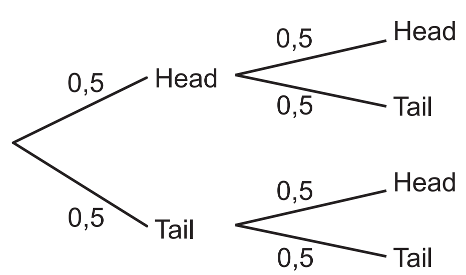
21. Show learners that the answer to the second question could have been calculated by taking the probability of pulling out a red T-shirt away from 1. The reason is that there were only two options available – getting a red T-shirt or not getting a red T-shirt.

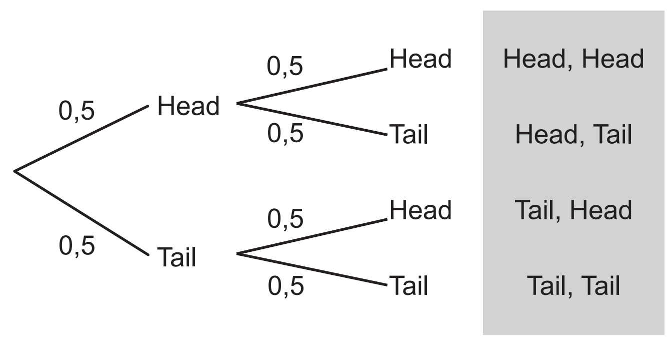
$$1-\frac{1}{5}=\frac{4}{5}$$

22. Tell learners that this idea will be expanded on in a later lesson.

23. Give the following scenario to learners: *I am going to toss a coin twice. Let's draw a tree diagram to represent all the possible outcomes and all the probabilities.*
    Ask: *What are the outcomes of tossing a coin?* (Getting heads or tails – so two in total)

24. Draw the following tree diagram one piece at a time. Point out key issues to learners as you draw the tree diagram. .

	Teaching notes
	Start at a point and draw two branches representing the two possible outcomes.Write the outcomes at the end of the branches. Write the theoretical probability ON the branch. Tell learners that they must not deviate from these conventions.
	Extend the tree diagram to represent the second toss. Tell learners: *Even though there will only be one more toss, it needs to be represented twice – as if heads was tossed on the first throw and as if tails were tossed on the first throw – to cover ALL possibilities.*



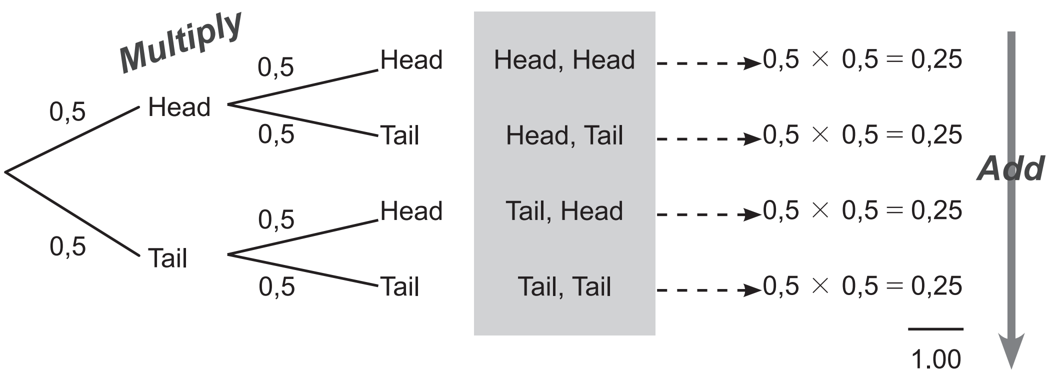
Write all the possible outcomes at the end of the branches.
Run your finger along the different branches to show all the possibilities.

25. Tell learners: *A tree diagram is a simple way of representing a sequence of events.*

26. Point out that no matter how many branches come from one point, the probabilities on those branches should always add up to one because all the possibilities are represented.

27. Ask: *Are these events dependent or independent?*
(Independent – how the coin landed the first time does not affect what will happen the second time).

28. Use this diagram (add onto your previous diagram) to show how probabilities are calculated using a tree diagram:



29. Ask learners to consider the probability of getting *at least one head.* First confirm the meaning of this statement: '*at least one head'*. In the two tosses, one head will be acceptable but so will two heads as this also covers the statement 'at least one'.

30. Tell learners to look at all the outcomes.
Ask: H*ow many of these outcomes have at least one head in them?*
(3 of them do. Only the last one – Tail, Tail - has no heads at all).

31. To calculate the probability of at least one head being tossed:
    - first we multiply along the branches that lead to each of these three outcomes
    - second we add all of those answers together.

$$\left(\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{2}\right)=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{3}{4}$$

32. Ask: *Describe a shorter way to find this probability (we discussed this earlier).*
    (1 subtract the probability of getting no heads at all as the two possibilities add up to 1 because those two outcomes – getting at least one head and getting no heads – represent all outcomes)

$$1-\frac{1}{4}=\frac{3}{4}$$

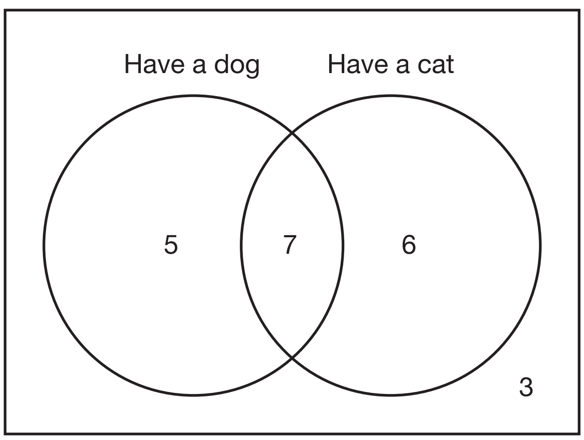
33. Summarise the following points and ask learners to write them in their books:
    - To find the probability of something happening <u>AND</u> something else happening, multiply the probabilities.
    - To find the probability of something happening <u>OR</u> something else happening, <u>add</u> the probabilities.

34. Draw this table on the chalkboard. The table represents the same situation as the tree diagram – the tossing of a coin twice:

		Coin 1	
		Head	Tail
Coin 2	Head	H-H	H-T
	Tail	T-H	T-T

35. Tell learners this is a two-way table. The outcomes are read by looking down the columns and across the rows. Note the four possible outcomes: H-H; H-T; T-H; T-T.

36. There is one other concept to cover before ending the revision lesson – Venn diagrams. Draw this example on the chalkboard:



37. Explain: *A Venn diagram is a useful way to represent the relationships between two or more sets. The overlapping circles show what is shared.*
38. Ask: *What information is represented in this Venn diagram?*
    (Responses to the question of whether you own a dog or a cat)

    Ask: *How many people owned a cat?*
    (7 + 6 = 13)

    Ask: *How many people owned a dog?*
    (7 + 5 = 12)

    Ask: *How many people owned neither a dog nor a cat?*
    (3)

    Ask: *How many people owned both a dog and a cat?*
    (7)
39. Tell learners that they will be learning more about Venn diagrams this term.
40. Ask directed questions so that you can ascertain learners' level of understanding.
    Ask learners if they have any questions.
41. Give learners an exercise to complete with a partner.
42. Walk around the classroom as learners do the exercise. Support learners where necessary.

D

ADDITIONAL ACTIVITIES/ READING

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=bh3yZuVSUC4
(What is probability)

https://www.youtube.com/watch?v=vGcmjINp1x8
(Probability word problems)

Term 4, Topic 1, Lesson 2

VENN DIAGRAMS

Suggested lesson duration: 2,5 hours

POLICY AND OUTCOMES

A

CAPS Page Number	29
Lesson Objectives By the end of the lesson, learners should be able to: • use Venn diagrams to calculate probabilities • derive and explain the identity: $P(A\ or\ B) = P(A) + P(B) – P(A\ and\ B)$.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson list the three sets.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
2	300	2 (Leave out 3c)	299	15.1	256	17.2 17.3 17.5	381 385 391	14.3 14.4 14.5	479 482 484

C

CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. It is important for learners to have a good understanding of Venn diagrams. Learners need to be able to draw Venn diagrams from given information and to read information correctly from Venn diagrams.
2. Take care to ensure each learner's understanding – walk around the classroom to check on learners' work and encourage them to ask questions.

DIRECT INSTRUCTION

1. Start the lesson with the following sets listed on the board:

   Say: *Consider the set of whole numbers from 1 to 10*:

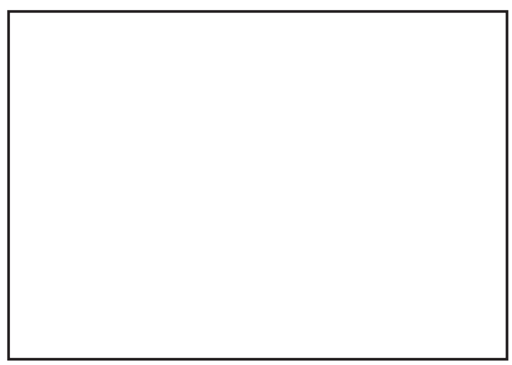
   $\{1;2;3;4;5;6;7;8;9;10\}$

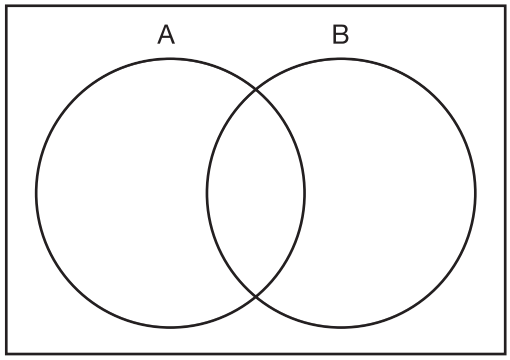
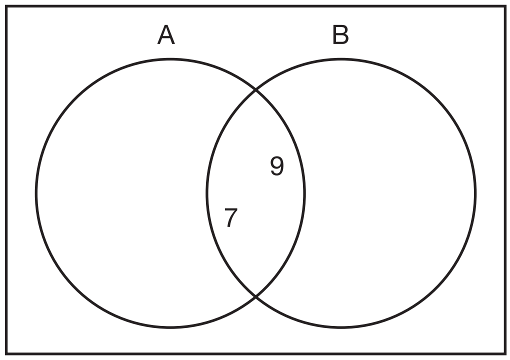
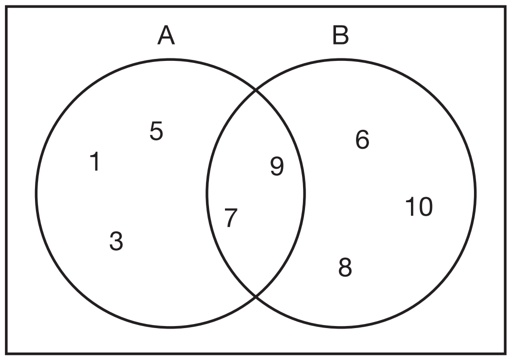
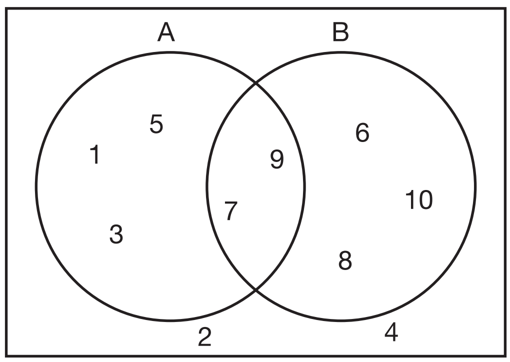
   We will define two sets taken from this group of numbers:

   Set A = the odd numbers in the group = {1, 3, 5, 7, 9}

   Set B = the numbers which are 6 or more in the group = {6, 7, 8, 9, 10}

2. Say: *This is an ideal situation to represent in a Venn diagram. Let's draw one together.*

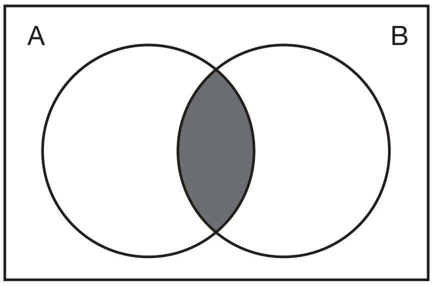
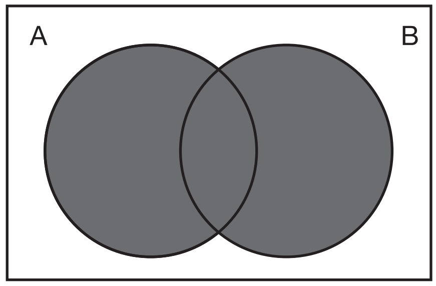
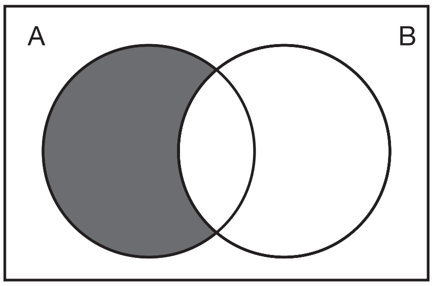
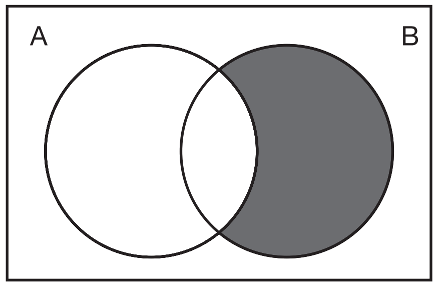
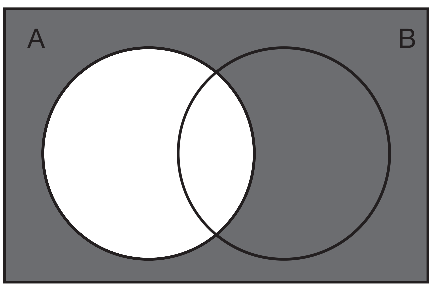
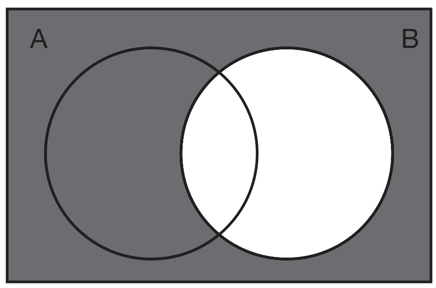
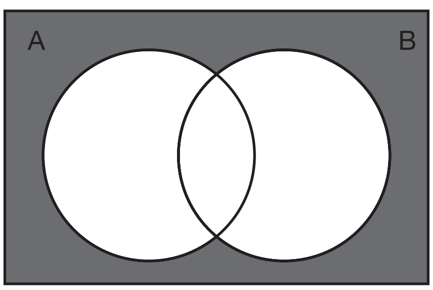
	Say:	**Do:**
Step 1	Draw a frame to represent the sample space. This is important as it will represent the entire sample space – in this case, the set of whole numbers from 1 to 10. When the diagram is complete, all 10 numbers should have been used.	

Step 2 **Step 3**	There are two sets. Draw two overlapping circles inside the sample space frame. Label the sets (circles) A and B.	
Step 4	Always start with the common elements of the two sets. Consider the numbers that belong in both set A and set B. Write the common numbers in the overlapping part – the intersection.	
Step 5 **Step 6**	Complete set A by writing the other (three) numbers into set A. Complete set B by writing the other (three) numbers into set B.	
Step 7	Complete the diagram by writing any left-over numbers from the sample space which did not belong in either of the two sets.	

Ensure learners can see where you get the numbers from to fill in onto the diagram.
As you fill in each number, cross it out from the set list.

3. Use the diagram to discuss the terms: intersection, union, A only, B only, not A, not B and not A or B.

   As each term is mentioned point out or tick the area on the Venn diagram, then draw a sketch representing the term by shading the appropriate area.
   Learners should write these in their exercise books.

intersection	union
	
A only	B only
	
not A	not B
	
not (A or B)	
	

4. Explain:
   - intersection means AND – the sets must BOTH share the element
   - union means OR – either one could have the element

> Discuss this with learners.
>
> In general, rules of mathematics follow rules of life. For example, just as we can't add unlike terms in mathematics, we can't add them in real life either (desks and chairs). In probability, this may not seem the case. When a person is given an option – would you like to play a game <u>or</u> do work – they need to choose, and their options are limited. When a person is offered a biscuit <u>and</u> a juice they are receiving both – their options are not limited.
>
> In probability, it is the opposite of this – OR creates more options; AND limits the options.
>
> Use the following example to demonstrate this to learners.
>
> Say:
>
> *All the people who take History raise one hand.*
> (Learners should count/ estimate the number of hands).
>
> *Thank you – put your hands down now.*
>
> *All the people who take Geography raise one hand.*
> (Learners should count/ estimate the number of hands).
>
> *Thank you – put your hands down now.*
>
> *All the people who take History AND Geography raise one hand.*
>
> (Point out that there are fewer hands – the option was limited by the use of the word AND).
>
> *Thank you – put your hands down now.*
>
> *All the people who take History OR Geography raise one hand.*
>
> (Point out all the hands – now everyone who put their hands up for the first two questions should raise their hands again – OR doesn't matter if they do one or the other or both – they are still doing at least one which covers the 'OR').
>
> *Thank you – put your hands down now.*
>
> Write on the board:
>
> 'In probability <u>**OR**</u> is M<u>**OR**</u>E' for learners to take down in their books.

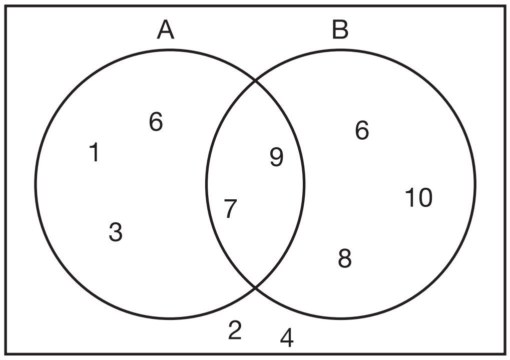
5. Use the table to discuss the symbols used for some of these terms.
   Add each symbol as you discuss it and tell learners to do the same.

intersection	union
$A \cap B$	$A \cup B$
A and B	A or B

not A	not B	not (A or B)
A'	B'	$(A \text{ or } B)'$

6. Explain to learners that an element is a member of a set. For example, 1 is an element of the set of odd numbers less than 10.

7. Use the Venn diagram from earlier to discuss further notation:



Complete a table such as the one below as the discussion progresses.

Notation	What does it mean?	Answer using the above Venn diagram
$n(S)$	The number of elements in the sample space, S.	10
$n(A)$	The number of elements in the set, A.	5
$n(B)$	The number of elements in the set, B.	5
$n(A\ or\ B)$ $n(A \cup B)$	The number of elements in the set, A or B.	8
$n(A\ and\ B)$ $n(A \cap B)$	The number of elements in the set, A and B.	2
$n(A')$	The number of elements NOT in the set, A.	5
$n(B')$	The number of elements NOT in the set, B.	5
$n(A\ or\ B)'$ $n(A \cup B)'$	The number of elements NOT in the set, A or B.	2
$n(A\ and\ B)'$ $n(A \cap B)'$	The number of elements NOT in the set, A and B.	8
Probability		
$P(S)$	The probability of an element being in the sample space	$\frac{10}{10} = 1$
$P(A)$	The probability of an element being in set A	$\frac{5}{10}$
$P(B)$	The probability of an element being in set B	$\frac{5}{10}$

$P(A\ or\ B)$	The probability of an element being in the set, A or B.	$\frac{8}{10}$
$P(A\ and\ B)$	The probability of an element being in the set, A and B.	$\frac{2}{10}$
$P(A')$	The probability of an element NOT being in set, A.	$\frac{5}{10}$
$P(B')$	The probability of an element NOT being in set, B.	$\frac{5}{10}$
$P(A\ or\ B)'$	The probability of an element NOT being in set, A or B	$\frac{2}{10}$
$P(A\ and\ B)'$	The probability of an element NOT being in the set, A and B.	$\frac{8}{10}$

The fractions for the probability answers have not been simplified for a reason. It is important that learners have a clear understanding where the numbers came from. The denominator is always the total in the sample space and the numerator is always the number of elements in the set concerned.

8. Use the same Venn diagram to discuss the probability of A or B happening.

   Show how this probability could be found:

   Say: *We could find the probability of A* (point to the full circle representing the set A) *then add that to the probability of B* (point to the full circle representing B), *but then what have we done that we will need to undo?*

   (The probability of the intersection has been counted twice so we need to subtract it).

   This will only be easy for learners to see if you have used your finger and hand to show that you took the whole of A and then the whole of B.

   Therefore, a formula for the probability of A or B is:

   $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

   Ask if everyone can see why this will always work.

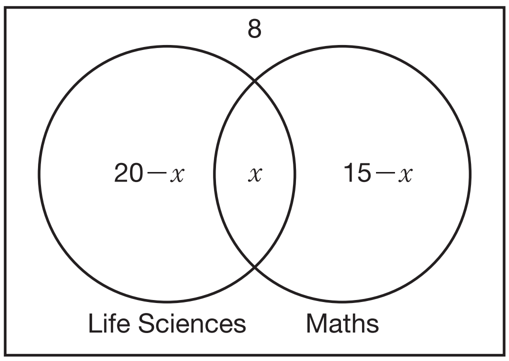
   Answer any questions learners may have.

   Tell learners to write it in their books and highlight it in some way to make it stand out.

9. Do the following example with learners. Learners should write it in their books.

Example	Teaching notes
There are 40 learners in a class: 15 do Mathematics; 20 do Life Sciences and 8 do neither. Find the number of learners who do both subjects.	Tell learners that drawing a Venn diagram will make this question easier. Tell them to draw the basis of a Venn diagram (2 circles overlapping and the frame representing the sample space). Ask: *Where should you always start?* (The intersection). Ask: *What makes this difficult?* (The number of learners who do both is not given). Say: *It is still essential to start with the intersection, therefore we need to use a variable to represent the unknown.* Ask: *If we call the number of learners doing both subjects* x *– how many learners do mathematics?* ($(15-x)$ as the amount represented by x have already been counted and need to be deducted from the 15). Ask: *How many learners do LS?* $(20-x)$. Tell learners to complete their Venn diagram.

Solution:



Ask: *How many learners are there altogether?*

(40)

Say: *Make an equation of all the learners represented adding up to make 40.*

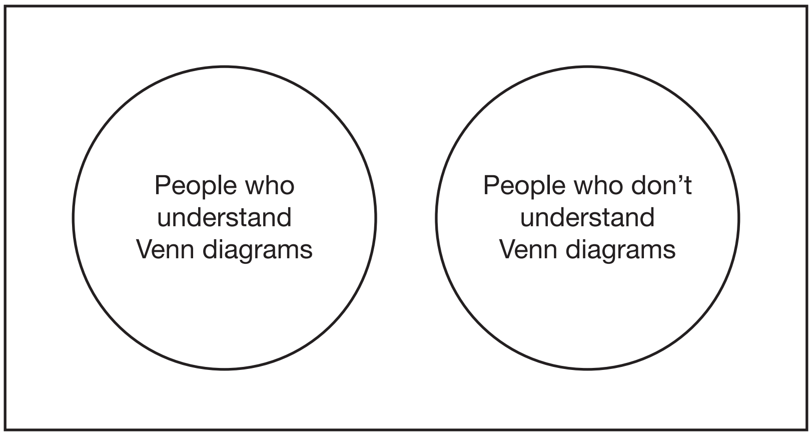
$$20-x+x+15-x+8=40$$
$$43-x=40$$
$$-x=-3$$
$$\therefore x=3$$

10. Ask directed questions so that you can ascertain learners' level of understanding.
    Ask learners if they have any questions.

11. Give learners an exercise to complete with a partner.

12. Walk around the classroom as learners do the exercise. Support learners where necessary.

13. When learners have completed the exercise and it has been corrected, draw this Venn diagram on the board for a little light relief (and learning):



Ask, while pointing at the circle on the left: *Who belongs here?*

(Hopefully everyone will raise their hand!)

Ask: *Why is there no intersection?*
(Either you understand Venn diagrams, or you don't – there is no overlapping).

Tell learners that we will look at this idea a little further in the next lesson.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.youtube.com/watch?v=z5Gc1R3EbZs
(Introduction)

https://www.youtube.com/watch?v=MSw0JDbfNhA
(Venn diagram notation)

https://www.youtube.com/watch?v=xwK--rNDI9E
(Examples)

Term 4, Topic 1, Lesson 3

INCLUSIVE AND MUTUALLY EXCLUSIVE EVENTS; COMPLEMENTARY AND EXHAUSTIVE EVENTS

Suggested lesson duration: 2,5 hours

A POLICY AND OUTCOMES

CAPS Page Number	29
Lesson Objectives By the end of the lesson, learners should be able to: • differentiate between inclusive events and mutually exclusive events • use the rules related to mutually exclusive events • define and give examples of complementary and exhaustive events • use the rules related to complementary and exhaustive events • use their knowledge of complementary and exhaustive events to calculate probabilities.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. Draw the Venn diagram for point 1, as well as the outlines for the two examples in point 4. Once you have discussed the examples for point 4, leave them on the board as they will be discussed again in point 10.
5. The table on the next page provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.

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LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
3 4	303 308	3c	299	15.2	260	17.6	395	14.6 14.7	486 487

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

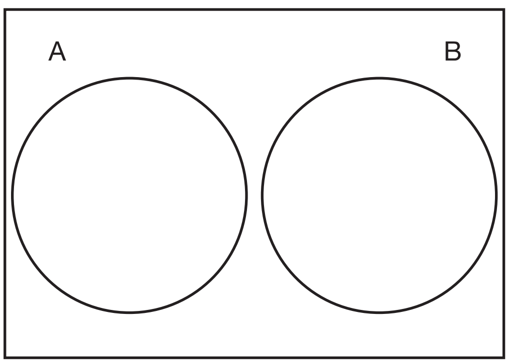
1. The terms covered in this lesson are regularly used in probability. Learners need to understand their meaning to make sense of a situation given in a question.
2. The words are not common words used in everyday language. For this reason you need to use the terminology correctly yourself and also quiz the learners often to confirm they know the meanings.

DIRECT INSTRUCTION

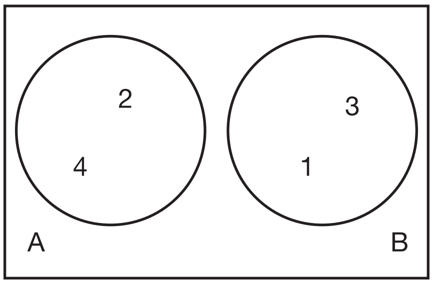
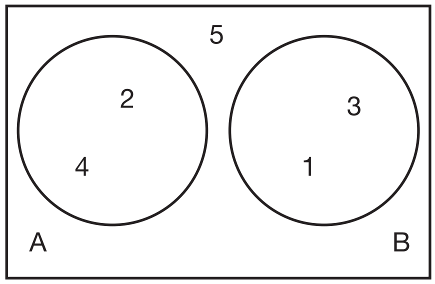
1. Start the lesson by asking: *Is it possible for a learner to be in Grade 10 and Grade 11 at the same time?*

   (No).

   Ask: *If a Venn diagram was used to represent learners in Grade 10 and Grade 11, what would it look like?*
   (It would have no intersection).
2. The Venn diagram would look like this:



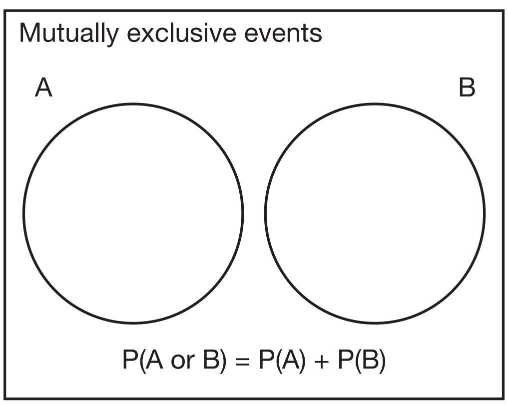
3. Say: *These events are called mutually exclusive events – they do not share any elements*. Tell learners to write the heading, ‘Mutually exclusive’ in their books and draw the diagram underneath it, making a note that the key idea is ‘no intersection’.
4. Point out that the situation of mutually exclusive events could be shown in two types of Venn diagrams:

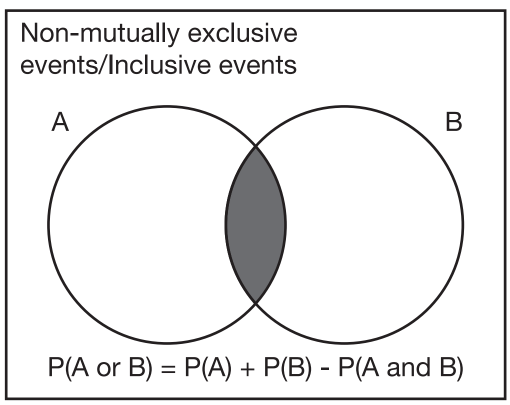
	
Sample space: the natural numbers less than 5. Set A: even numbers Set B: odd numbers	Sample space: the natural numbers from 1 to 5 Set A: even numbers less than 5 Set B: odd numbers less than 5

5. Discuss the difference:
   - In the first example, there are no elements left over as all elements have been used in the sets.
   - In the second example, there is one element left over (5).

   However, both examples represent mutually exclusive events.
6. Tell learners to write the two examples, as well as the full definition of mutually exclusive events, in their exercise books.

   Definition of mutually exclusive events: Events with no outcomes in common (no intersection). Events that cannot happen at the same time.
7. Tell learners that when events do have an intersection, they are called *non-mutually exclusive events* or *inclusive events*.
8. Summarise the concepts of mutually exclusive events and non-mutually exclusive (inclusive) events for learners using probability notation:





Tell learners to write this in their exercise books.

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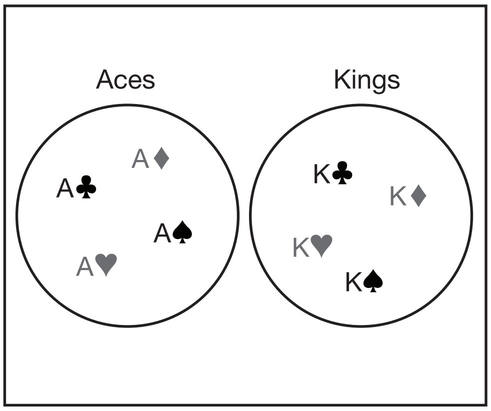
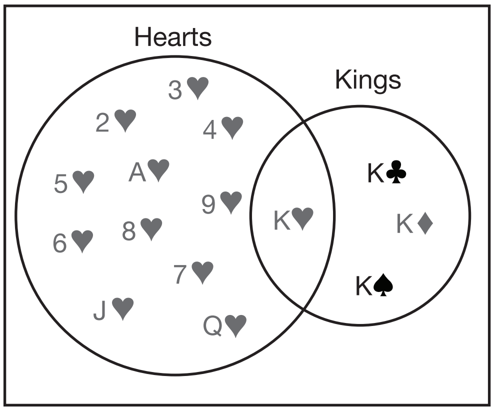
9. Ensure that learners are clear as to why we do not subtract the intersection for mutually exclusive events (it is equal to zero).

10. If learners are familiar with playing cards, this example explains mutually exclusive and non-mutually exclusive (inclusive) clearly:



Aces and Kings are mutually exclusive (can't be both)



Hearts and Kings are not mutually exclusive (can be both)

11. Focus learners' attention on the two examples used in point 4. Ask learners to remind you what the difference was between them (in one example all the elements were used and in the other all the elements were not used).

12. Ask: *Do you agree that in the first example, if one set occurred it would be impossible for the other set to occur?* (Yes).

    Ask: *Would you also agree that at least one of the sets had to occur?* (Yes).

13. Say: *When this is the case, the events are called complementary events – mutually exclusive events that have only two possible outcomes.*

14. Give the following examples:

    Girls and boys would create mutually exclusive events (no intersection) that are complementary events - if one event occurs (girls) the other cannot occur (boys).

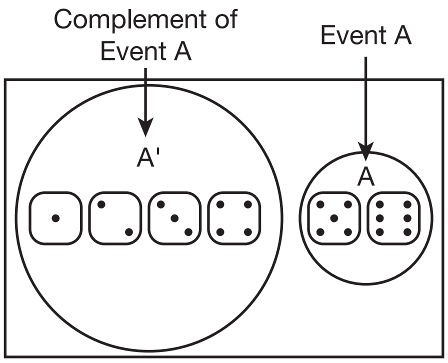
    If there is a 20% chance it will rain today, there must be an 80% chance that it will not rain. These are complementary events – when one occurs the other cannot occur.

15. Tell learners to write the heading, 'Complementary events' in their books.

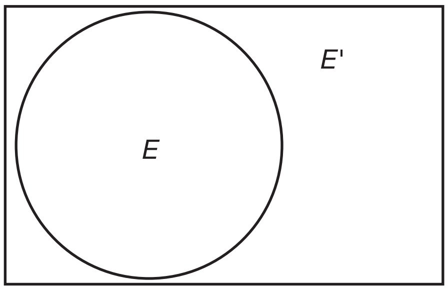
16. Write the definition on the board for learners to take down:

    Mutually exclusive events that contain all the outcomes between them. They are the only two possible outcomes.

17. Use the example of throwing a die to illustrate further. Explain that when the die is thrown, you would like to get a 5 or a 6. Draw this diagram on the board:



18. Show learners the notation used to show complementary events using the following diagram to illustrate:



Explain: The complement of event E is shown as E' – this means the set of all outcomes in the sample space that are not in event E.

$$P(E)+P(E)'=1$$

19. Tell learners to copy the Venn diagram with the rule into their books. Once they have completed this, extend the rule further:

$$P(E)=1-P(E)'$$

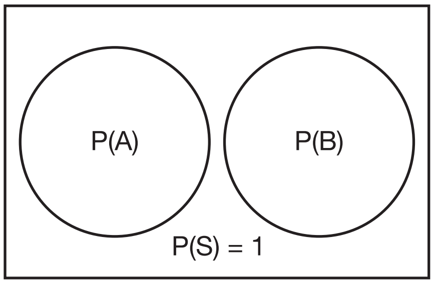
OR $$P(E)'=1-P(E)$$

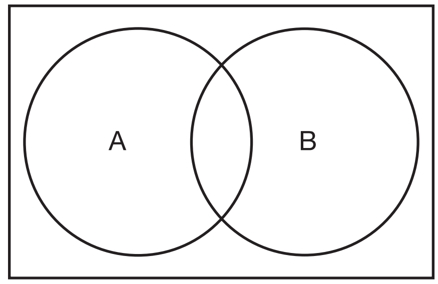
20. Tell learners: *The final concept we are going to learn about today is 'exhaustive events'.*

21. Ask: *Do you agree that when you have used all your energy, you could say that you are exhausted?*

The word has a similar meaning in probability – if all the elements in the sample space are used then we say the events are exhaustive.

22. Draw the following Venn diagrams to explain further:





Point out to learners that both situations represent exhaustive events even though one has an intersection and the other doesn't.
The key aspect is that there are no left-over elements in the sample space: all elements are in one set or the other.

23. Ask learners to write the heading 'Exhaustive events' in their books and draw these two Venn diagrams.

24. Write the definition on the board for learners to take down:

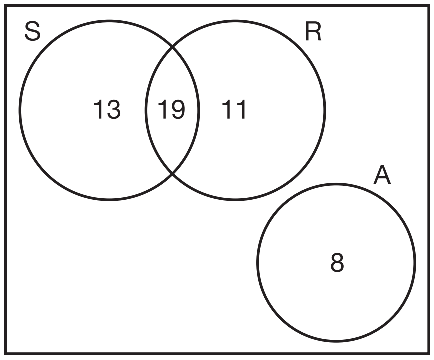
    The union of two events is equal to the entire sample space.

25. Show learners the notation used to show exhaustive events:

$$P(A \text{ or } B) = 1$$

$$P(A \cup B) = 1$$

26. Ask if anyone has any questions. Do the following example with learners:

<table>
<tr><th>Example</th><th>Teaching notes</th></tr>
<tr><td>Fifty learners were asked which of the following sports they enjoyed watching on television: soccer, rugby, athletics.
19 said soccer and rugby
11 said rugby only
8 said athletics
32 said soccer.
The information is represented in the Venn diagram below:

</td><td></td></tr>
<tr><td>a) Which events are mutually exclusive? Justify your answer.
b) Which events are inclusive? Justify your answer.
c) Are any of the events complementary? Justify your answer.
d) If one learner is chosen at random, what is the probability that he/she enjoys watching athletics or rugby?</td><td>Discuss each of these questions with learners. Ensure they give reasons for their answers.
For d), the probability question, remind learners to first ensure they know the total, then focus on the learners of interest – rugby or athletics.</td></tr>
<tr><td colspan="2">Solutions:
a) Soccer and athletics; rugby and athletics. There are no learners that like to watch both soccer and athletics or both rugby and athletics. (There is no intersection)
b) Soccer and rugby. There are learners who like to watch both. (There is an intersection).
c) None of the events are complementary. No matter which pair of sports you look at, there are always some learners not accounted for. For example, soccer and rugby are not complementary events due to the 8 learners who like to watch athletics. Similarly, for rugby and athletics (13 learners are not accounted for) as well as for soccer and athletics (11 are not accounted for).
d) $\frac{38}{51} = 0,75$</td></tr>
</table>

27. Ask directed questions so that you can ascertain learners' level of understanding.
    Ask learners if they have any questions.

28. Give learners an exercise to complete with a partner.

29. Walk around the classroom as learners do the exercise. Support learners where necessary.

ADDITIONAL ACTIVITIES/ READING

D

Further reading, listening or viewing activities related to this topic are available on the following web links:

https://www.mathsisfun.com/data/probability-events-mutually-exclusive.html
(Explanation of mutually exclusive)

Term 4, Topic 1, Lesson 4

REVISION AND CONSOLIDATION

Suggested lesson duration: 1,5 hours

A

POLICY AND OUTCOMES

CAPS Page Number	29
Lesson Objectives By the end of the lesson, learners will have revised and consolidated: ● all concepts covered in previous lessons.	

B

CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation: Work through the lesson plan and exercises.
3. Write the lesson heading on the board before learners arrive.
4. Write work on the chalkboard before the learners arrive. For this lesson have the first question written up.
5. The table below provides references to this topic in Grade 10 textbooks. Plan when you will get learners to practice the concepts learned by completing the exercises. Work through the lesson plan and decide where you will get learners to do the exercises. Indicate this on your lesson plans.
6. In addition to the exercise in your textbook, a past test may also be worthwhile.

LEARNER PRACTICE

MIND ACTION SERIES		PLATINUM		SURVIVAL		CLASSROOM MATHS		EVERYTHING MATHS (SIYAVULA)	
EX	PG	EX	PG	EX	PG	EX	PG	EX	PG
Rev Some Ch	310 311	Rev	300	W/sh	263	17.7 17.8	398 400	14.8	490

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

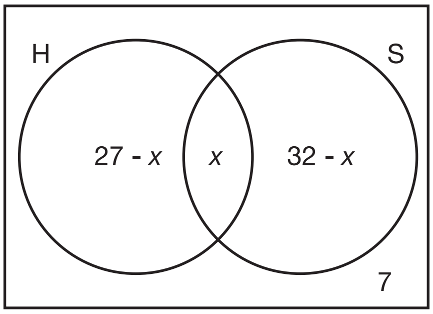
1. Ask learners to recap what they have learned in this section. Spend time pointing out issues that you know are important as well as problems that you encountered from your own learners.
2. If learners want you to explain a concept again, do that now.

DIRECT INSTRUCTION

This lesson is made up of fully worked examples from a past examination covering most of the concepts in this topic. As you work through these with the learners, it is important to frequently talk about as many concepts as possible.
For example, use the words complementary, mutually exclusive, union, intersection etc wherever possible, constantly reminding learners what they have already learnt.

Say: *I am going to do an entire Probability question from the 2016 final examination with you. You should write them down as I do them, taking notes at the same time.*

FETMaths_Term4_Gr10_LP.indb 31 2018/07/30 12:08:05 PM

<table>
<tr><th>Example</th><th>Teaching notes</th></tr>
<tr><td>
In a certain class of 42 boys:
● 27 play hockey (H)
● 32 play soccer (S)
● 7 do not play hockey or soccer
● An unknown number play both hockey and soccer (x)
The information is represented in the Venn diagram below:


a) Calculate the value of x
b) If a boy from the class is chosen at random, calculate the probability that he:
(i) does not play hockey or soccer
(ii) plays only soccer
</td><td>
First discuss the Venn diagram with learners to ensure they are clear why x represents the intersection and is therefore required when representing hockey only and soccer only.

a)
Ask: what is the total number of boys? (42)
Say: therefore, the numbers represented in the Venn diagram should total 42. Make an equation.
b)
Tell learners to fill in the actual numbers now that x has been found.
</td></tr>
<tr><td colspan="2">
Solution:
a) $27 - x + x + 32 - x + 7 = 42$
$66 - x = 42$
$-x = -24$
$x = 24$
b) (i) P (not play hockey or soccer) $= \frac{7}{42} = \frac{1}{6}$
(ii) P (soccer only) $= \frac{8}{42} = \frac{4}{21}$
</td></tr>
<tr><th>Example</th><th>Teaching notes</th></tr>
<tr><td>
A bag contains 3 blue balls and x yellow balls.
a) Write down the total number of balls in the bag
b) If a ball is drawn from the bag, write down the probability that it is blue
</td><td>
Tell learners not to be put off by the variable. Ask: what would you do if the question had said, 2 yellow balls? (add 3 and 2, therefore we should add 3 and x)
</td></tr>
<tr><td colspan="2">
Solution:
a) $x + 3$
b) $\frac{3}{x+3}$
</td></tr>
</table>

<table>
<tr><th>Example</th><th>Teaching notes</th></tr>
<tr><td>a) Complete the following statement: If A and B are two mutually exclusive events, then $P(A\ and\ B) = \ldots$
b) Given that A and B are mutually exclusive events. The probability that event A occurs is 0,55. The probability that event B does not occur is 0,7. Calculate $P(A\ or\ B)$.</td><td>a)
Ask: What does mutually exclusive mean?
(no intersection)
b)
Ask: What statement can be made from the identity learned about $P(A\ or\ B)$, keeping in mind that there is no intersection?
$(P(A\ or\ B) = P(A) + P(B))$
Say: But to use this, we need P(B). Note the words, ‘does not occur’ – what do they imply?
(complementary events).</td></tr>
<tr><td colspan="2">Solution:
a) $P(A\ and\ B) = 0$
b) $P(B) = 1 - P(B')$
$= 1 - 0,7$
$= 0,3$
$P(A\ or\ B) = P(A) + P(B)$
$= 0,55 + 0,3$
$= 0,85$</td></tr>
</table>

1. Ask directed questions so that you can ascertain learners’ level of understanding.
   Ask learners if they have any questions.

2. Give learners an exercise to complete on their own.

3. Walk around the classroom as learners do the exercise. Support learners where necessary.

Term 4

REVISION OVERVIEW

A TOPIC OVERVIEW

- The revision plan runs for four weeks (18 hours).
- The revision plan is not presented over specific lessons. We provide guidance as to what to do each week. Plan according to your own learners needs.
- Learners will write two examinations in November. Each examination is two hours and 100 marks. Encourage learners to be well prepared.
- It is essential that these revision weeks are used well. Learners need to feel confident when writing their final examinations.
- The revision programme is made up of three parts:
  - Summary notes to share with learners
  - Paper 1 and Paper 2 (2017) to work through with learners in detail
  - Paper 1 and Paper 2 (exemplars) for learners to work on in class and at home and make 'cheat sheets' (their own summaries) at the same time.

Breakdown of revision programme:

Week 1	Paper 1 summary notes and past Paper 1
Week 2	Paper 2 summary notes and past Paper 2
Week 3& 4	Paper 1 and Paper 2 + 'cheat sheets'

As part of the revision programme, learners will work through past papers. This has been shown to be an excellent learner-centred approach to revision. In addition to providing the past papers and memoranda in the Resource Pack, we also provide the following links:

Links for past papers and memoranda:

Paper 1 2017	http://www.edwardsmaths.com/wp-content/uploads/2018/01/NSC-GR10-MATHS-P1-NOV2017-ENG.pdf
Paper 1 2017 memo	http://www.edwardsmaths.com/wp-content/uploads/2018/01/NSC-GR-10-Maths-P1-Memo.pdf
Paper 2 2017	http://www.edwardsmaths.com/wp-content/uploads/2018/01/NSC-GR10-MATHS-P2-NOV2017-ENG.pdf

Paper 2 2017 memo	http://www.edwardsmaths.com/wp-content/uploads/2018/01/Final-Marking-Guideline-Grade-10-Mathematics-P2-2017-Common-Examination.pdf
Paper 1 Exemplar 2012	http://maths.stithian.com/Gr10%20NEW%20CAPS%20Exemplars/Mathematics-P1-GR-10-Exemplar-2012-Eng.pdf
Paper 1 Exemplar 2012 memo	http://maths.stithian.com/Gr10%20NEW%20CAPS%20Exemplars/Mathematics-P1-GR-10-Exemplar-2012-Memo-Eng.pdf
Paper 2 Exemplar 2012	http://maths.stithian.com/Gr10%20NEW%20CAPS%20Exemplars/Mathematics-P2-GR-10-Exemplar-2012-Eng.pdf
Paper 2 Exemplar 2012 memo	http://maths.stithian.com/Gr10%20NEW%20CAPS%20Exemplars/Mathematics-P2-GR-10-Exemplar-2012-Memo-Eng.pdf

WHAT EXPERIENCE AND RESEARCH TELL US ABOUT PREPARING FOR EXAMINATIONS

What is the best way to revise for a mathematics exam?

- Do practice questions.
- Check your answers.
- Learn your theory.
- Focus on what you can do, as well as what you can't do.
- Discuss questions and methods with fellow learners. Explain to each other – this is an excellent way to consolidate your own understanding.
- Make a revision plan and stick to it.

Keep your eye on the prize – share these tips with your learners

Revising for mathematics exams can be hard work – it means making sacrifices where you need to prioritise revision over other things. Therefore, it is important to keep your eye on the prize. Think about what your maths qualification will mean for your future life and career. Hopefully this will keep you motivated when times are tough during revision.

ASSESSMENT

- CAPS formal assessment requirements for Term 4:
  - Test (already completed)
  - Final examinations (Paper 1 and Paper 2)
- The examinations will be made up as follows:

Paper 1

	Mark allocation
Number patterns	(15 ± 3)
Algebraic Expressions, Equations and Inequalities	(30 ± 3)
Functions	(30 ± 3)
Finance and Growth	(10 ± 3)
Probability	(15 ± 3)

Paper 2

	Mark allocation
Trigonometry	(40 ± 3)
Analytical geometry	(15 ± 3)
Euclidean Geometry and Measurement	(30 ± 3)
Statistics	(15 ± 3)

Term 4

REVISION – WEEK 1

POLICY AND OUTCOMES

A

CAPS Page Number	29
Lesson Objectives By the end of the lesson, learners will have: ● received summaries of all Paper 1 topics ● completed a full Paper 1 in class with their teacher.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation:
   - work through the summaries of Paper 1
   - work through the exam (2017) and teaching notes.
3. The notes and examination are both available in the Resource Pack (Resources 2 and 3) for photocopying if possible.
4. Write work on the chalkboard before the learners arrive to ensure no time is wasted.

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. It is important that you support learners as they consolidate all that they have learned this year.
2. Learners need to have time to ask questions and become confident in their ability to write their final examination.

FETMaths_Term4_Gr10_LP.indd 37 2018/07/30 12:08:09 PM

DIRECT INSTRUCTION

1. Start by handing out the six sets of summary notes for Paper 1 topics – Resource 2 in the Resource Pack.
2. Work through the notes with learners. This will take at least an hour.
3. As you go through each topic, ask questions to ascertain how much learners remember.
4. Encourage learners to add their own notes to the summary notes you have given them.
5. Once you have revised each section (for Paper 1), hand out the past examination paper (This is Resource 3 in the Resource Pack – Paper 1, 2017). Work though each question in detail. Some learners may be sufficiently confident to work on their own, while others may prefer to work with you.
6. As you go through each question, give learners the opportunity to contribute and ask questions.
7. Encourage learners to use their summary notes – for answering questions and to add their own notes as they go along.

ALGEBRAIC EXPRESSIONS

Given: $q = \sqrt{b^2 - 4ac}$

a) Determine the value of q if $a = 2$, $b = -1$ and $c = -4$. Leave your answer in simplest surd form.

b) State whether q is rational or irrational

c) Between which TWO consecutive integers does q lie?

Teaching notes:

a) This is a straightforward substitution question. Say: *Read the question carefully and note that the answer must be left in simplest surd form. Do not write it as a decimal.*

b) Ask: *What is a rational number?* (A number that can be written in the for $\frac{a}{b}$ where a and b are integers and $b \neq 0)$.

Ask: *What is an irrational number?* (A number that cannot be expressed as a common fraction).

c) Say: *This requires knowledge of square numbers and how to find the two perfect squares that lie on either side of the integer in the root sign.*

Solutions:

a) $q = \sqrt{b^2 - 4ac}$

$q = \sqrt{(-1)^2 - 4(2)(-4)}$

$q = \sqrt{33}$

b) Irrational

c) 5 and 6

Factorise the following expressions fully:

a) $t^2(r-s)-r+s$

b) $\dfrac{x^3+1}{x^2-x+1}$

Teaching notes:

Ask: *What do we always do first when asked to factorise?*

(Count the terms and look for a common factor).

a) *How many terms in this expression?* (Three)
   What do you notice about the last two terms? (They are similar to the two terms in the bracket and if -1 is taken out as a factor a common expression will be found in the two terms).

b) Ask: *What do you notice about the numerator?* (It is a sum of two cubes that can be factorised).

Solutions:

a) $t^2(r-s)-r+s$
$= t^2(r-s)-(r-s)$
$=(r-s)(t^2-1)$
$=(r-s)(t-1)(t+1)$

b) $\dfrac{x^3+1}{x^2-x+1}$
$=\dfrac{(x+1)(x^2-x+1)}{x^2-x+1}$
$=x+1$

Simplify the following completely:

a) $(2y+3)(7y^2-6y-8)$

b) $\dfrac{3}{x^2-9}+\dfrac{3}{(x-3)^2}$

c) $\dfrac{3^t-3^{t-2}}{2.3^t-3^t}$

Teaching notes:

a) Remind learners that to simplify means to use the distributive law to multiply and then collect like terms. Simplifying is the inverse of factorising. When simplifying we expect more than one term in our answer (although one term is possible); whereas to factorise, we expect one term only and there are usually brackets in the answer. This should be a straightforward question where the distributive law is used and like terms are collected.
   Say: *Knowing the rules of exponents is important.*

b) Ask: *What is the rule for adding or subtracting fractions?*
   (Find the lowest common denominator).
   Ask: *In algebraic fractions, what needs to be ensured first?*
   (That the denominators are fully factorised).
   Ask: *What do we need to do?*
   (Factorise the difference of two squares in the first fraction. The other denominator is fully factorised).

c) Remind learners how important it is to be able to 'undo' an exponential statement in order to make it easier to recognise the highest common factor as well as to know what is left over. Write the following on the board: $a^2 \times a^3 =$
Ask: *What is the answer?*
(a^5)
How did you get the '5'?
(By adding the exponents).
Write the following on the board: a^{2+3}
Ask: *If you went back to the step of what this looked like before it was simplified, what would it look like?*
($a^2 \times a^3$)
Repeat with a different example: 3^{a+3} (previous step was $3^a \times 3^3$)
Say: *Remember that this is what needs to be done in this question in order to take out a common factor. Remember the importance of knowing your exponential rules!*

Solutions:

a) $(2y+3)(7y^2-6y-8)$
$= 14y^3 - 12y^2 - 16y + 21y^2 - 18y - 24$
$= 14y^3 + 9y^2 - 34y - 24$

b) $\dfrac{3}{x^2-9} + \dfrac{2}{(x-3)^2}$
$= \dfrac{3}{(x+3)(x-3)} + \dfrac{2}{(x-3)^2}$
$= \dfrac{3(x-3)+2(x+3)}{(x+3)(x-3)^2}$
$= \dfrac{3x-9+2x+6}{(x+3)(x-3)^2}$
$= \dfrac{5x-3}{(x+3)(x-3)^2}$

c) $\dfrac{3^t - 3^{t-2}}{2.3^t - 3^t}$
$= \dfrac{3^t - 3^{t-2}}{2.3^t - 3^t}$
$= \dfrac{3^t - 3^t.3^{-2}}{2.3^t - 3^t}$
$= \dfrac{3^t(1-3^{-2})}{3^t(2-1)}$
$= \dfrac{(1-3^{-2})}{(2-1)}$
$= \dfrac{1-\frac{1}{9}}{1}$
$= \dfrac{8}{9}$

EQUATIONS, INEQUALITIES AND WORD PROBLEMS

a) Given $4 - 2x < 16$ where $x \in R$
  (i) Solve the inequality
  (ii) Hence, represent your answer to a)(i) on a number line
b) Solve simultaneously for x and y:
  $-2x - y = 10$ and $3x - 4y = -4$
c) Solve for x:
  (i) $\dfrac{x(x-5)}{6} - 1 = 0$
  (ii) $c = \sqrt{a+2x}$
d) Tabelo is currently four times as old as his daughter, Linda. Six years from now, Tabelo will be three times as old as Linda. Calculate Linda's age currently.

Teaching notes:

a) Remind learners that a linear inequality is treated the same as an equation, UNLESS we need to divide by a negative integer, then the inequality sign will change.
Ask: *How do we represent an inequality on a number line?*
If learners struggle to answer, go through the section in their summary notes on representing inequalities. Remember this is a learning experience so don't just stick to what is required for this particular question.

b) Ask: *How do we solve simultaneous equations?*
(Get one variable on its own in one equation, substitute the new information into the 2nd equation and solve for the unknown. Use this result to substitute back into the 1st equation and solve for the second variable).
Remind learners of the link to Functions: Say: *When you are solving simultaneous equations, you are actually finding the point(s) of intersection of two functions. These functions are both linear (straight lines) and therefore you will only find one set of solutions.*

c) Remind learners that inverse operations are necessary to solve for the given variable. We need to get the variable by itself. In (i) there is a fraction.
Say: *Remember that it is possible to not work with fractions at all when dealing with equations. We can find the LCD and multiply each term with it in order to have no fractions left.*
In (ii) we are dealing with literal equations.
Say: *Remember that the focus is on getting x alone. Use inverse functions.*
Ask: What is the inverse operation to square rooting? (Squaring).

d) Say: *It is best to deal with age problems by drawing a table. The table needs two rows to represent the people and two columns to represent the two different times (now and the other one mentioned). Mark the youngest person x and the oldest person in terms of x according to the information given. Use these ages to fill in the other time mentioned. In this case, the 'now' ages will need 6 years adding on.*
Once this has been done, remind learners we need to change the information into an equation.
Ask: *What comparison was given between the two ages in 6 years' time?*
(Tabelo will be three times as old)
Whose age will need to be multiplied by 3 to make the ages equal?
(Linda's).

Solutions:

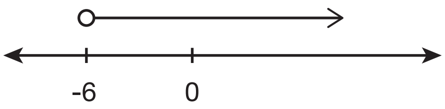
a) (i) $4-2x<16$

$$-2x<12$$

$$\frac{-2x}{-2}<\frac{12}{-2}$$

$$x>-6$$

(ii)



b) $-2x-y=10$ (1)

$3x-4y=-4$ (2)

$$-y=10+2x$$

$$y=-10-2x$$

Substitute into (2)

$$3x-4y=-4$$

$$3x-4(-10-2x)=-4$$

$$3x+40+8x=-4$$

$$11x=-4-40$$

$$11x=-44$$

$$x=-4$$

Substitute into (1)

$$-2x-y=10$$

$$-2(-4)-y=10$$

$$8-y=10$$

$$-y=2$$

$$y=-2$$

c) (i) $\frac{x(x-5)}{6}-1=0$ (LCD = 6)

$$x(x-5)-6=0$$

$$x^2-5x-6=0$$

$$(x+1)(x-6)=0$$

$x=-1$ or $x=6$

(ii) $c=\sqrt{a+2x}$

$$c^2=a+2x$$

$$c^2-a=2x$$

$$\frac{c^2-a}{2}=x$$

d)

	Now	6 years' time
Tabelo	$4x$	$4x+6$
Linda	x	$x+6$

$$4x+6=3(x+6)$$

$$4x+6=3x+18$$

$$4x-3x=18-6$$

$$x=12$$

Linda is currently 12 years old.

NUMBER PATTERNS

a) Consider the linear sequence: $5;8;11;b;17\ldots$
   (i) Write down the value of b.
   (ii) Determine the n^{th} term of the sequence.
   (iii) Calculate the value of the 15th term of the sequence.
   (iv) Which term in the sequence is equal to 83?

b) Consider the number pattern below created by using the numbers of the sequence $2;6;10;14;18;\ldots$

$$\begin{array}{ccccccccc} & & & & 2 & & & & \\ & & & 6 & & 10 & & & \\ & & 14 & & 18 & & 22 & & \\ & 26 & & 30 & & 34 & & 38 & \\ 42 & & \ldots & & \ldots & & \ldots & & \ldots \end{array}$$

   (i) Calculate the sum of the numbers in the 8th row.
   (ii) Determine the mean of the numbers in the 20th row.

Teaching notes:

a) No calculation is required to find b (i). The common difference is 3, therefore this can just be added to 11.

In order to find the nth term (ii) we need to know:

- what a linear sequence looks like in standard form $T_n = pn + q$
- what the variables represent.

  Ask: *What does the* p *represent?* (The common difference). Say: *Once this has been substituted, use the 1st term (5) to find* q *by making n=1 and asking what needs to be added to get 5.*

  The final two parts of the question require substitution:

- The first part – to find the 15th term will require substituting 15 into n's place.
- The second part – to find which term is 83, requires substituting T_n with 83 and solving for n.

b) Learners need to think beyond the regular linear pattern ideas (although the pattern of numbers is linear – it is set out in different rows and therefore the focus needs to be on the row). Learners need to look at the sum on each row and consider the new pattern formed to assist in answering the questions. The pattern formed by the sum of each row is:

2 ; 16 ; 54 ...

Give learners time to discuss what a general term (formula) could be for this pattern.

Hint for learners: Each number is even.

Ask: W*hat do we multiply by 2 to get each number?*

Ask: *Do you recognise these numbers?*

$$2 = 2 \times 1; 16 = 2 \times 8; 54 = 2 \times 27\ldots$$

1, 8 and 27 are the cube numbers! This should lead to the rule and to answer (i), substitute 8 for n.

Now that the ground work has been done, (ii) should be a little easier.

Substitute 20 for n but divide by 20 to find the mean.

Solutions:

a) (i) Constant difference $= 3$

$\therefore b = 14$

(ii) $T_n = pn + q$

$T_n = 3n + q$

$T_n = 3n + 2$

(iii) $T_n = 3n + 2$

$T_{15} = 3(15) + 2$

$T_{15} = 47$

(iv) $T_n = 3n + 2$

$83 = 3n + 2$

$81 = 3n$

$27 = n$

b) (i) $T_n = 2n^3$

$T_n = 2(8)^3 = 1024$

(ii) $T_n = 2(20)^3 = 16\ 000$

Mean: $\frac{16000}{20} = 800$

FINANCE AND GROWTH

a) Seven years ago Mrs Grey decided to invest R18 000 in a bank account that paid simple interest at 4,5% p.a.

(i) Calculate how much interest Mrs Grey has earned over the 7 years.

(ii) Mrs Grey wants to buy a television set that costs R27 660.00 now. If the average rate of inflation over the last 5 years was 6,7% p.a., calculate the cost of the television set 5 years ago.

(iii) At what rate of simple interest should Mrs Grey have invested her money 7 years ago if she intends buying the television set now using only her original investment of R18 000 and the interest carried over the last 7 years?

b) On a certain day the exchange rate between the US dollar and South African rand is \$1 $=$ R12,91. At the same time the exchange rate between the British pound and the South African rand is £1 $=$ R16,52.

Calculate the exchange rate between the British pound and US dollar on that day.

Teaching notes:

a) Note: For (i) *Interest, not the interest rate, is required. The interest is an amount of money. For (ii)* Ask: *What kind of interest is used for inflation?* (Compound interest).
For (iii), point out that this will require finding the rate of interest. This requires substituting the known values then solving an equation.
Ask: *What can you use for 'A' and 'P' as you have not been given the amount that is being saved?*
Variables are preferable but even 2 and 1 could be used or 20 and 10 etc. Any of these could be used.

b) As the rate has been given in one unit for both the dollar and the pound, we can compare the two-rand values to find the ratio.
Learners may prefer to get both exchange rates given to R1.
The dollar and pound can then be compared.

Solutions:

a) (i) $A = P(1 + i.n)$

$A = 18000(1 + (0{,}045)(7))$

$A = 23670$

$\therefore$ Interest earned is: $\mathrm{R}23670 - \mathrm{R}18000 = \mathrm{R}5670$

(ii)

$$A = P(1+i)^n$$
$$27660 = P(1+0{,}067)^5$$
$$\frac{27660}{(1+0{,}067)^5} = P$$
$$20000 = P$$

(iii)

$$A = P(1 + i.n)$$
$$27660 = 18000(1 + i(7))$$
$$\frac{27660}{18000} = 1 + 7i$$
$$\frac{27660}{18000} - 1 = 7i$$
$$\frac{\frac{27660}{18000} - 1}{7} = i$$
$$i = 0{,}076666\ldots.$$

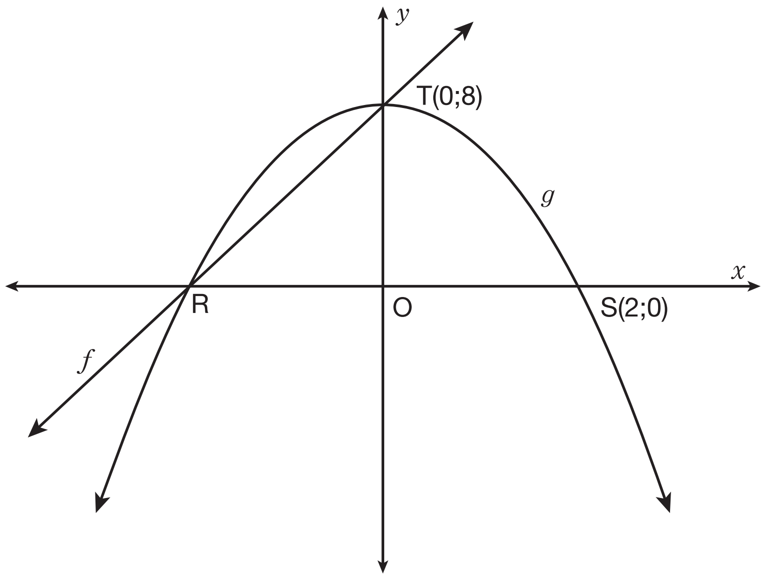
$\therefore$ Mrs Grey needed an interest rate of 7,67%.

b) $\dfrac{pound}{dollar} = \dfrac{16{,}52}{12{,}91}$

$\therefore$ £$1 = \$1{,}28$

FUNCTIONS

The diagram shows the graphs of $g(x) = ax^2 + q$ and $f(x) = mx + c$.
R and S(2; 0) are the x-intercepts of g and T(0; 8) is the y-intercept of g.
Graph f passes through R and T.



a) Write down the range of g.
b) Write down the x-coordinate of R.
c) Calculate the values of a and q.
d) Determine the equation of f.
e) Use the graphs to determine the value(s) of x for which:
   (i) $f(x) = g(x)$
   (ii) $x.g(x) \leq 0$
f) The graph h is obtained when g is reflected along the line $y = 0$. Write down the equation h in the form $h(x) = px^2 + k$.

Teaching notes:

a) Ask: *What values are related to the range?*
(All the possible y-values)

b) Remind learners that the x-intercepts are always the same distance from the axis of symmetry of a parabola which, in this case, is the y-axis. Learners can therefore count.

c) Learners need to know what the variables a and q represent.
Ask: *What is the value of q?*
(8 as it represents the vertical shift). Once one value is known any other point on the function can be substituted to find the 2nd variable.

d) Say: *Remember when looking for the equation of the function – which will always require finding the values of variables – there will always be at least the number of pieces of information to match the number of variables.*
Ask: *In this case, what needs to be found?*
(m and c).
Ask: *What information do we have?*
(Two points – the x and y-intercepts). Remind learners they need to know what the variables represent so they can substitute c with the y-intercept then use the other point to find m.

e) The first part of the question (i) should be straightforward. Learners need to look for the points of intersection. The second part needs to be done in a few steps.
Say: *First consider the algebra.*
Ask: *What kind of numbers need to be multiplied to give a negative number?*
(One negative number and one positive number).
Say; *This means that when x is negative, the function ($g(x)$) needs to be positive. When x is positive, the function ($g(x)$) needs to be negative.*
Once the values have been identified, remind learners to consider the 'or equal to zero' part to know what to include. Remind learners it is important to know how to represent inequalities.

f) Draw a sketch of a Cartesian plane on the board and plot the coordinate.
(1, 3)
Ask: *If this point was reflected in the x-axis, what would the new point be?*
$(1;-3)$
Ask; *If this point was reflected in the y-axis, what would the new point be?*
$(-1; 3)$
Ask learners to look carefully at what happened to the coordinates.
Reflection in the x-*axis:* $(x;y) \rightarrow (x;-y)$
Reflection in the y-*axis:* $(x;y) \rightarrow (-x;y)$
Remind learners that all they need to do when asked to reflect in a function in one of the axes is perform the rule accordingly.
Ask: *What axis is the line $y = 0$?*
(The x-axis).

Solutions: a) $y \leq 8$ b) The x-coordinate of R is -2 c) $g(x) = ax^2 + q$ $q = 8$ $g(x) = ax^2 + 8$ $(2;0)$ $0 = a(2)^2 + 8$ $0 = 4a + 8$ $-8 = 4a$ $\therefore a = -2$	d) $f(x) = mx + c$ $f(x) = mx + 8$ $(-2;0)$ $0 = m(-2) + 8$ $2m = 8$ $m = 4$ $\therefore f(x) = 4x + 8$ e) (i) $x = -2; x = 0$ (ii) $x.g(x) \leq 0$ $x \in [-2;0]$ or $x \in [2;\infty)$ f) $g(x) = -2x^2 + 8$ $h(x) = -(-2x^2 + 8)$ $h(x) = 2x^2 - 8$

The function $p(x) = k^x + q$ is described by the following properties:

- $k > 0; k \neq 1$
- $x-$intercept at (2; 0)
- The horizontal asymptote is $y = -9$

a) Write down the range of p.
b) Determine the equation of p
c) Sketch the graph of p. Show clearly the intercepts with the axes and the asymptote.

Teaching notes:

a) Remind learners that the range is directly affected by the horizontal asymptote – that is where this exponential function starts but does not include the point at the horizontal asymptote of $y = -9$.
Ask: *Is this function above or below the asymptote?*
(Above because $k > 0$)
b) Remind learners that there are two variables required, therefore there should be two pieces of information.
Ask: *What information is given?*
(The asymptote which also represents the vertical shift and a coordinate. Once the vertical shift has been substituted (q), the point can be used to find the other variable (k).
c) Remind learners to work neatly and to mark all appropriate points.

Solutions:

a) $y > -9$

b) $p(x) = k^x + q$

$p(x) = k^x - 9$

$(2;0)$

$0 = k^2 - 9$

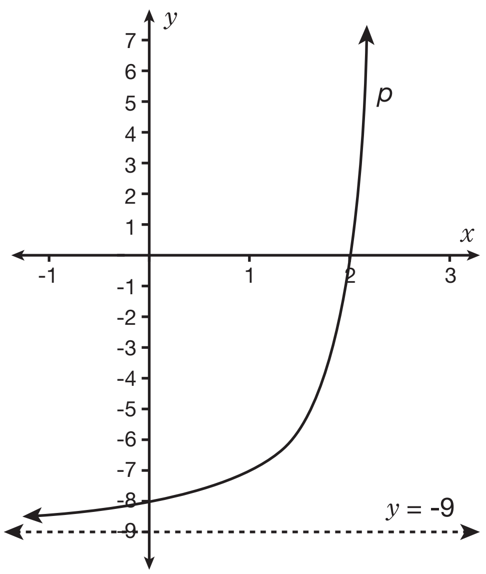
$0 = (k+3)(k-3)$

$k = -3$ or $k = 3$

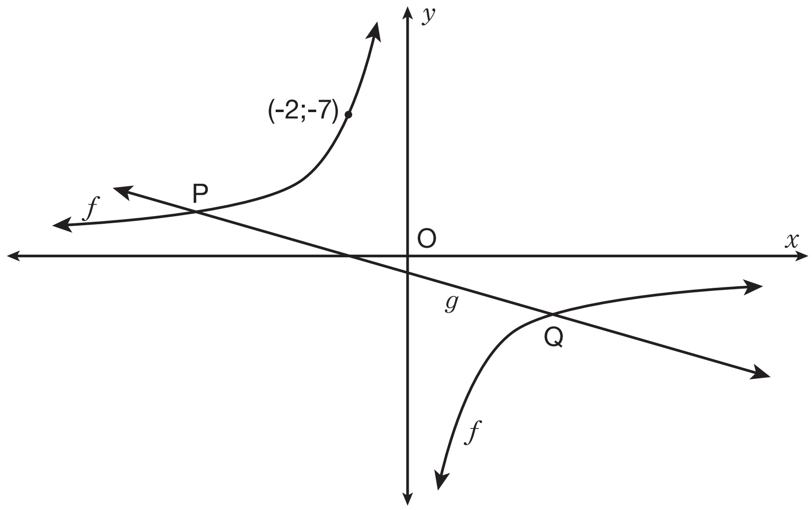
$\therefore k = 3$

$\therefore p(x) = 3^x - 9$

c)



The sketch below shows the graphs of $f(x) = \frac{k}{x} + w$ and $g(x) = -x - 1$.
The graph of g is an axis of symmetry of f. The graphs f and g intersect at P and Q.



a) Write down the value of w.

b) The point $(-2;7)$ lies on f. Calculate the value of k.

c) Calculate the $x-$coordinates of P and Q.

d) Write down the values of x for which $\frac{-16}{x} > -x$

Teaching notes:

a) Ask: *What does the variable 'w' represent?*
(The vertical shift and hence the horizontal asymptote).
Point out that the axis of symmetry will assist in answering this question. Remind learners that the axis of symmetry of a hyperbola always passes through the points where the two asymptotes meet. Ask: *What is the equation of the vertical asymptote?*
$(x = 0)$
Ask: What is the equation of the horizontal asymptote?
$(y = -1)$ (because that is the y-intercept of the straight line).

b) Once w has been found, this is a straightforward substitution of the point given.

c) Ask: *What is happening at P and Q?*
(The points of intersection of the two graphs).
Ask: *How do we find the points of intersection?*
(Make the equations equal and solve for both variables).

d) This will not be an easy question. Many learners might struggle with where to get started. Tell learners to start by looking carefully at what has been given.
Ask: *What connection do the functions given have to the functions we have been working with?* Learners need to notice that they are dealing with the functions both moving one unit up (vertical shift). Hence the -1 in both functions is now zero.
Learners therefore need to shift the graphs up (this is probably the best way for them to see it visually). Once that has been done, learners should use their ruler placed parallel to the y-axis to run along the x-axis and to look where the hyperbola is bigger than (above) the straight line. Remind learners to check whether zero is included. In this case it is not (<).

Solutions:

a) $w = -1$

b) $f(x) = \frac{k}{x} + w$

$f(x) = \frac{k}{x} - 1$

$(-2;7)$

$7 = \frac{k}{-2} - 1$

$8 = \frac{k}{-2}$

$-16 = k$

c) $f(x) = g(x)$

$\frac{-16}{x} - 1 = -x - 1$ LCD $= x$

$-16 - x = -x^2 - x = 0$

$x^2 - 16 = 0$

$(x+4)(x-4) = 0$

$\therefore x = -4$ or $x = 4$

$x_P = -4$ and $x_Q = 4$

d) $\frac{-16}{x} - 1 = -x - 1$

$\therefore \frac{-16}{x} = -x$

$\therefore \frac{-16}{x} > -x$

$-4 < x < 0$ or $x > 4$

PROBABILITY

Two events, A and B are complementary and make up the entire sample space.
Also, $P(A') = 0{,}35$.

a) Complete the statement: $P(A)+P(B)=$...
b) Write down the value of $P(A\ and\ B)$
c) Write down the value of $P(B)$.

Teaching notes:

This question relies on a good theoretical knowledge of probability.

a) Ask: *What does complementary mean?*
   (Two outcomes in which the two outcomes are the only possible outcomes).
   Ask: *This then means that the probability of one outcome added to the probability of the other outcome must be equal to….?*
   (1).
   Learners may want to write the addition rule if they know it and this would be marked correct.
b) Ask: *Do complementary events have an intersection?*
   (No – if one event is happening the other one cannot be happening as well).
c) Ask: *What does $P(A')$ mean?*
   (The probability of 'not A').
   Ask: *As these events are complementary, that must mean that $P(A')$ must be the same as…?*
   $P(B)$.

Solutions:

a) $P(A)+P(B)=1$ or $P(A)+P(B)=P(A\ or\ B)$
b) $P(A\ and\ B)=0$
c) $P(B)=P(A')=0{,}35$

A survey was conducted among 150 learners in Grade 10 at a certain school to establish how many of them owned the following devices: smartphone (S) or tablet (T).
The results were as follows:

- 8 learners did not own either a smartphone or a tablet
- 20 learnes owned both a smartphone and a tablet
- 48 learners owned a tablet
- learners owned a smartphone.

a) Represent the above information in a Venn diagram.
b) How many learners only owned a smartphone?
c) Calculate the probability that a learners selected at random from this group:
   (i) owned only a smartphone
   (ii) owned at most only one type of device.

Teaching notes:

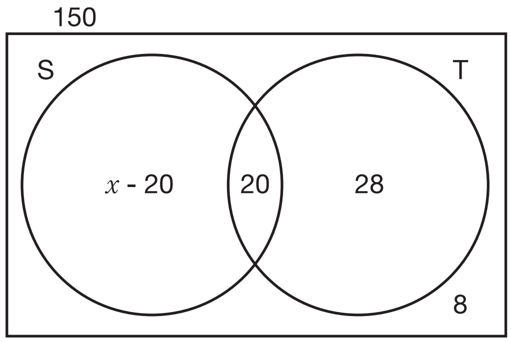
a) Ask: *How many events are represented*?
(2)
Where should we always start when completing a Venn diagram?
(The intersection)
How many learners owned both a smartphone and a tablet? (20)
Remind learners that they need to subtract in order to fill in the rest of the events represented by the circles as the 20 learners in the intersection have already been counted for both the smartphones and the tablets.
Tell learners not to be put off by an unknown value, but to use it in the same way they would use an actual value.

b) Ask: *How will we find the unknown value?*
(Make the sum of all values equal to the total 150 and solve). Remind learners that this is not the answer to the question.
Ask: *What needs to be done once the unknown value is found?*
(Subtract 20).

c) Ask: *What total will be used to find probability?*
(150)
Ask: *How many learners owned only a smartphone?*
(94)
The second part of the question needs to be discussed in more detail:
Learners must be clear what they are looking for. The words 'at most' and 'at least' are often used in probability questions. Mathematically, 'at least' is the same as greater than or equal to and 'at most' is the same as less than or equal to. Use the words in an everyday sentence to explain to learners. For example, I want you to get at least 70% for this test. Ask: *What marks would be acceptabl*e?
(70%, 85%, 98% etc).
Learners should note that the 'at least' actually means that particular number or more.
Another example: You should spend at most 1 hour on your phone a day.
Ask: *What length of time would be acceptable?*
(1 hour, 35 minutes, 20 minutes etc).
Therefore, <u>at most only one type of device</u> means that they could own only one type or none at all. Ask: *Which learners are represented by this?*(94 – smartphone only; 28 – tablet only and 8 – none at all).
Point out that as this is everyone except the 20 learners represented in the intersection, the answer could also be found by saying 1 subtract the probability of the intersection.

Solutions:

a)



b) $x - 20 + 20 + 28 + 8 = 150$

$x = 114$

$\therefore$ smartphone only: $114 - 20 = 94$

c) (i) $P(only\, S) = \frac{94}{150} = 0,63$

(ii) $1 - P(A\ and\ B) = 1 - \frac{20}{150}$

$= \frac{130}{150}$

$= \frac{13}{15}$

$= 0,87$

8. When the past paper has been completed, ask learners if they have any questions.

9. Say: *Next week we will be revising the work for Paper 2.*

Term 4

REVISION – WEEK 2

A POLICY AND OUTCOMES

CAPS Page Number	29
Lesson Objectives By the end of the lesson, learners will have: ● received summaries of all Paper 2 topics ● completed a full Paper 2 with the teacher.	

B CLASSROOM MANAGEMENT

1. Make sure that you are ready and prepared.
2. Advance preparation:
   - work through the summaries of Paper 2
   - work through the exam (2017) and teaching notes.
3. The summary and examination for paper 2 are available in the Resource Pack (Resources 4 and 5) for photocopying if possible.
4. Write work on the chalkboard before the learners arrive to ensure no time is wasted.

C CONCEPTUAL DEVELOPMENT

INTRODUCTION

1. It is important for learners to consolidate all they have learned this year.
2. Learners need to have time to ask questions and become confident in their ability to write their final examination.

DIRECT INSTRUCTION

1. Start the lesson by handing out the four sets of summary notes for Paper 2 topics available in the Resource Pack (Resource 4).
2. Go through the notes with learners. This should take at least an hour.
3. Ask questions to ascertain how much learners remember as you go through each topic.
4. Encourage learners to add their own notes to the summaries – now and throughout the next few weeks of revision.
5. Once each section has been covered, hand out the past examination paper (Resource 5). Do each question in detail with learners. Allow learners who feel confident to work on their own to do so.
6. As you go through each question, give learners the opportunity to contribute and ask questions.
7. Encourage learners to refer to their summary notes and to use them when answering questions or to add notes to if they are finding something a challenge.

STATISTICS

The data below shows the number of laptops sold by 15 sales agents during the last financial year

43	48	62	52	46	90	58	37	48	73	84	68	54	34	78

a) Determine the median number of laptops sold.
b) Calculate the range of data.
c) Calculate the interquartile range.
d) Draw a box and whisker diagram for the data above.

Teaching notes:
a) Remind learners that to find the median, the data needs to be ordered.
b) Largest value subtract smallest value.
c) Find the upper quartile and lower quartile and subtract the lower quartile from the upper quartile.
d) Use the five-number summary and remember to make sure the scale is accurate.

Rearrange the data in ascending or descending order:

34 37 43 46 48 48 52 54 58 62 68 73 78 84 90

In this example, the data has been arranged in ascending order.

a) Median

$$\frac{1}{2}(n+1) = \frac{1}{2}(15+1)$$
$$= \frac{1}{2}(16)$$
$$= 8$$

The median is in the 8th position. $\therefore$ the median is 54

b) $90 - 34 = 56$

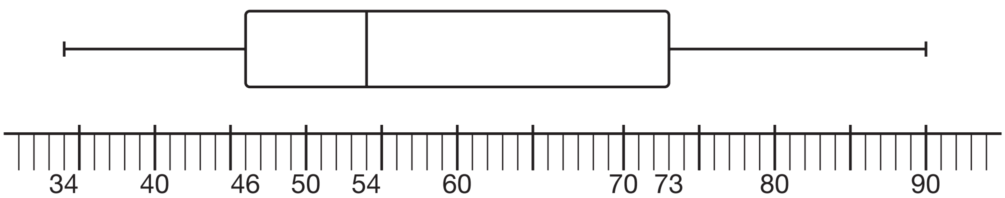
c) $Q_3 - Q_1$

$= 73 - 46$

$= 27$

d) Five-number summary:

34 46 54 73 90 (90 – 34 = 56. Suggested scale: 1cm = 5 units)



A learner did a project on climate change. At 14:00 each day, she recorded the temperature (in °C) for a certain town. The information is given in the frequency table below.

Temperature (in ºC)	Frequency
$20 \le T < 24$	2
$24 \le T < 28$	4
$28 \le T < 32$	9
$32 \le T < 36$	5
$36 \le T < 40$	7
$40 \le T < 44$	3

a) For how many days did the learner collect data?

b) Write down the modal class for the data.

c) Estimate the mean of data.

d) Calculate the percentage of days on which the temperature was at least 28°C.

Teaching notes:

Advise learners to look carefully at the information and summarised data to ensure it makes sense to them.

a) Ask: *Where will the information of number of days be read?*
(In the frequency column).

b) Ask: *What is a modal class?*
(The class with the highest frequency).

c) Ask: *Why is an estimate of the mean being asked for?*
(The exact mean is not possible as we don't know the exact temperatures for each day).
Ask: *How do we find estimated mean?*
(Find the midpoint of each interval and multiply by the frequency. Total the values and divide by 30).

d) Ask: *What does 'at least mean'?*
(Greater than or equal to). Tell learners they need to find this value then find the percentage out of 30 days.

Solutions:

a) 30 days

b) $28 \leq T < 32$

c) Estimated mean:

$$\frac{2(22)+4(26)+9(30)+5(34)+7(38)+3(42)}{30}$$

$$= \frac{44+104+270+170+266+126}{30}$$

$$= \frac{980}{30}$$

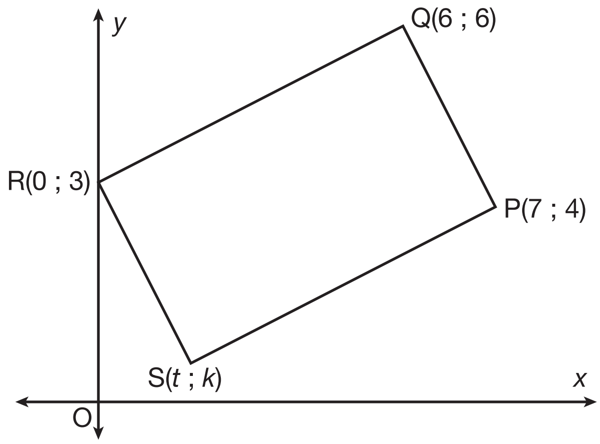
$$= 32,67^{\circ}\mathrm{C}$$

d) Total days: $9+5+7+3=24$

$$\frac{24}{30} \times 100 = 80\%$$

ANALYTICAL GEOMETRY

In the diagram below, $\mathrm{P}(7;4), \mathrm{Q}(6;6), \mathrm{R}(0;3)$ and $\mathrm{S}(t;k)$ are the vertices of a quadrilateral PQRS



a) Calculate the length of PQ. Leave your answer in surd form.
b) If $\mathrm{T}\left(\frac{7}{2};\frac{7}{2}\right)$ is the midpoint of QS, determine the coordinate of S.
c) If the coordinates of S are $(1;1)$, show that $\mathrm{PR} = \mathrm{QS}$.
d) Show that $\mathrm{QR} \perp \mathrm{RS}$
e) Hence, what type of quadrilateral is PQRS? Motivate your answer.
f) Calculate the size of $\mathrm{R\hat{S}Q}$.

Teaching notes:

a) This should be a straightforward distance answer, using the two points on the line segment and the distance formula.
b) Point out the word ‘midpoint’. This should immediately alert learners to the fact that they will use the midpoint formula.
Ask: *What makes this different to a regular midpoint question?*
(The midpoint is given, and the second coordinate is required. The formula will be used in reverse).
c) Ask: *How will we answer this question?*
(Find the distance of both lines and hope that they are the same). Remind learners: if the question states ‘show that’ or ‘prove that’, then the information is true. Tell learners if they get different answers they have made a mistake and should check their working.
d) Ask: *How do we prove that two lines are perpendicular?*
(Find the gradient of each. The product of the two gradients should equal -1).
e) Point out the importance of knowing the properties of quadrilaterals and knowing how to prove each property. Refer to the summary in the notes if necessary.
f) As the previous question showed that PQRS was a rectrangle, this can now be assumed. By drawing in a diagonal in the rectangle, a right-angled triangle is formed. Trigonometry can be used to find the angle required. Remind learners again, that two topics can be, and often are, combined in assessments.

Solutions: a) $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $PQ = \sqrt{(7-6)^2 + (4-6)^2}$ $PQ = \sqrt{(1)^2 + (-2)^2}$ $PQ = \sqrt{5}$ b) $\frac{6+t}{2} = \frac{7}{2}$ $\frac{6+t}{2} = \frac{7}{2}$ $6 + t = 7$ $t = 1$ $\frac{6+k}{2} = \frac{7}{2}$ $\frac{6+k}{2} = \frac{7}{2}$ $6 + k = 7$ $k = 1 \therefore S(1;1)$ c) $PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $PR = \sqrt{(7-0)^2 + (4-3)^2}$ $PR = \sqrt{(7)^2 + (1)^2}$ $PR = \sqrt{50}$ $PR = 5\sqrt{2}$ $QS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $QS = \sqrt{(1-6)^2 + (1-6)^2}$ $QS = \sqrt{(-5)^2 + (-5)^2}$ $QS = \sqrt{50}$ $QS = 5\sqrt{2}$ $\therefore PR = QS$	d) $m_{QR} = \frac{6-3}{6-0}$ $m_{QR} = \frac{3}{6} = \frac{1}{2}$ $m_{RS} = \frac{3-1}{0-1}$ $m_{RS} = \frac{2}{-1} = -2$ $m_{QR} \times m_{QR} = -1$ $\therefore QR \perp RS$ e) PQRS is a rectangle. One of the angles is 90°C and the diagonals are equal. f) In ΔQRS: $QS = 5\sqrt{2}$ and $RS = \sqrt{5}$ (opposite sides of a rectangle) $\cos R\hat{S}Q = \frac{\sqrt{5}}{5\sqrt{2}}$ $\therefore R\hat{S}Q = 71{,}57^o$

TRIGONOMETRY

a) Given $4\cot\theta + 3 = 0$ and $0^o < \theta < 180^o$.

(i) Use a sketch to determine the value of the following. DO NOT use a calculator.

1. $\cos\theta$ 2. $\frac{3\sin\theta\sec\theta}{\tan\theta}$

(ii) Hence, show that $\sin^2\theta - 1 = -\cos^2\theta$

b) Simplify the following expression WITHOUT the use of a calculator:

$$\cos 30^o \tan 60^o + \operatorname{cosec}^2 45^o \sin^2 60^o$$

c) Solve for θ correct to TWO decimal places, if:

$\frac{4}{3}\sin\theta = \cos 37^o$ and $0^o \leq \theta \leq 90^o$

Teaching notes:

a) Remind learners how different the Pythagoras questions are in trigonometry.
The actual question (in this case $\cos\theta$) cannot be even considered unless all the groundwork has been done to find x, y and r.
Ask: *What is the first step?*
(To get the trig ratio on its own).
Ask: *What needs to be done next?*
(Use the two pieces of information to find which quadrant we need to work in and draw the triangle)
Ask: *Once the triangle has been drawn, what should be done?*
(The ratios can be filled in – on the length of sides and as a coordinate – then Pythagoras will be used to find the missing side)
Both parts of the question in (i) can now be answered through substituting the ratios and calculating.
(ii) Remind learners to work with one side only when proving an identity. Substitution using the values found in (a) will be used.

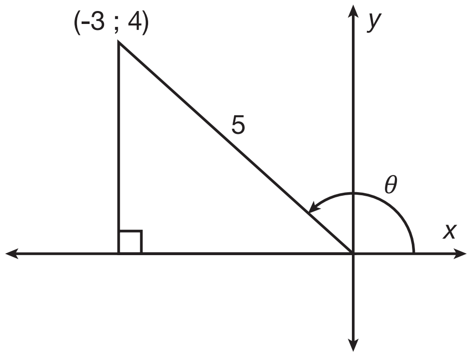
b) Remind learners that:
- a simplification without the use of a calculator will always require knowledge of special angles.
- the trig function of an angle is ALWAYS a ratio. It is good practice to open and close a bracket ready to insert the fraction (ratio) concerned for every trig function of the given angle.
- knowing the reciprocal trig functions is also important.

c) Ask: *How do we solve trig equations?*
(Get the trig function on its own; use the inverse of the trig ratio to find the angle).

Solutions:

a) (i)

$$4\cot\theta + 3 = 0$$
$$4\cot\theta = -3$$
$$\cot\theta = \frac{-3}{4}$$



1. $\therefore \cos\theta = -\frac{3}{5}$

2. $\frac{3\sin\theta\sec\theta}{\tan\theta} = \frac{3\left[\left(\frac{4}{5}\right)\left(-\frac{5}{3}\right)\right]}{\left(-\frac{4}{3}\right)}$

$$= \frac{-4}{-\frac{4}{3}}$$
$$= 3$$

(ii)

$$\text{LHS} = \sin^2\theta - 1$$
$$= \left(\frac{4}{5}\right)^2 - 1$$
$$= \frac{16}{25} - 1$$
$$= -\frac{9}{25}$$
$$\text{RHS} = -\cos^2\theta$$
$$= -\left(\frac{3}{5}\right)^2$$
$$= -\frac{9}{25}$$
$$\therefore \sin^2\theta - 1 = -\cos^2\theta$$

b) $\cos 30^o \tan 60^o + \operatorname{cosec}^2 45^o \sin^2 60^o$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{1}\right) + \left(\frac{2}{\sqrt{2}}\right)^2\left(\frac{\sqrt{3}}{2}\right)^2$$
$$= \frac{3}{2} + \left(\frac{4}{2}\right)\left(\frac{3}{4}\right)$$
$$= \frac{3}{2} + \frac{3}{2}$$
$$= 3$$

c) $\frac{4}{3}\sin\theta = \cos 37^o$

$$\frac{4}{3}\sin\theta = 0{,}79863551$$
$$4\sin\theta = 3(0{,}79863551)$$
$$\sin\theta = \frac{3}{4}(0{,}79863551)$$
$$\therefore \theta = 36{,}8^o$$

Given $f(x) = \sin x - 1$ and $g(x) = 2\cos x$ for $0^o \leq x \leq 270^o$.

a) Sketch the graph of f and g on the same axes for $0^o \leq x \leq 270^o$.

b) Write down the following:

(i) Amplitude of g

(ii) Range of f

c) Use your graph to determine the following:

(i) The number of solutions to $f(x) = g(x)$ in the interval $0^o \leq x \leq 270^o$

(ii) The value(s) in the interval $0^o \leq x \leq 180^o$ for which $\sin x = 2 + 2\cos x$

Teaching notes:

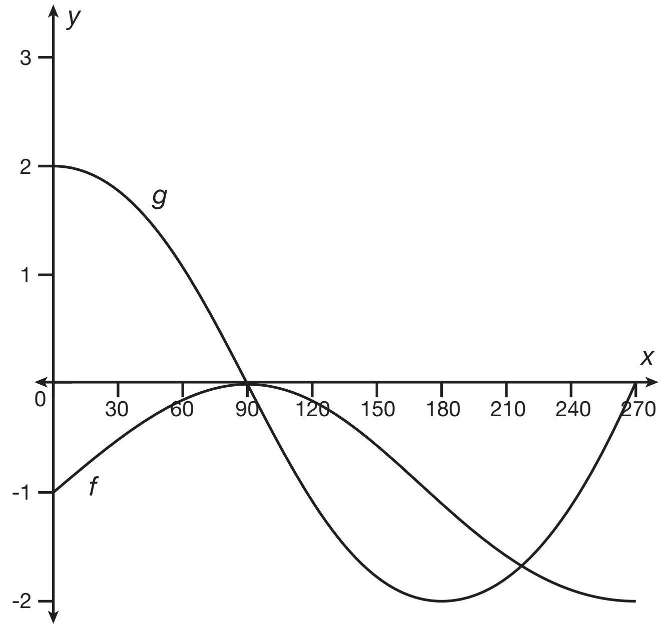
a) Remind learners that it is essential that they know the three basic trig graphs. They also need to understand what effect transformations a (amplitude) and q (vertical shift) have on a graph. This knowledge will assist them in transforming the basic function accordingly.
Ask: *What does the '-1' do to the sine graph?*
(Shift it down one unit).
Ask: *What does the '2' do to the cosine graph?*
(Stretch it from to -2 to 2).
Remind learners to take note of the domain given.

b) This question requires a theoretical knowledge and understanding of the vocabulary.
Ask: *What does the term amplitude mean?*
(The height from the mean of the function; or more simply, the height of the function will be two times the amplitude).
Ask: *What is the range?*
(All the possible output values of the function – in other words all the possible y-values).
Remind learners that the amplitude is only an integer whereas the range must be represented as an inequality or interval notation.

c) Once the graph has been drawn, it should be easy to see the number of solutions where the graphs are equal (the points of intersection).
The second part of this question is more difficult. Tell learners that they will need to manipulate the statement given to 'see' the original two functions. Once the functions are in their original form, they need to look at the equation again to see what it means.
When you have written $\sin x - 1 - 2\cos x = 1$ on the board:
Ask: *What does this mean?*
(Where is the difference between the sine graph and the cosine graph equal to 1)

Solutions:

a)



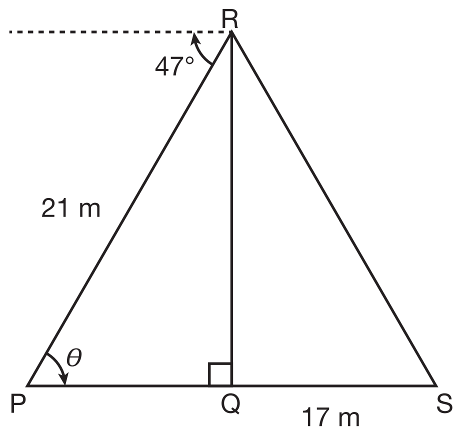
b) (i) Amplitude of g is 2

(ii) The range of f is $y \in [-2;0]$

c) (i) There are two solutions

(ii)
$$\sin x = 2 + 2\cos x$$
$$\sin x - 1 - 2\cos x = 1$$
$$f(x) - g(x) = 1$$
$$\therefore x = 126{,}87^{o} \text{ or } x = 180^{o}$$

RQ is a vertical pole. The foot of the pole, Q, is on the same horizontal plane as P and S. The pole is anchored with wire cables RS and RP. The angle of depression from the top of the pole to point P is 47°. PR is 21m and QS is 17m. $\mathrm{R\hat{P}Q} = \theta$.



a) Write down the size of θ.

b) Calculate the length of RQ.

c) Hence, calcualte the size of $\hat{\mathrm{S}}$.

d) If P, Q and S lie in a straight line, how far apart are the anchors of the wire cables?

Teaching notes:

It is important to look at each piece of information carefully and to link the information to the diagram.

RQ is a vertical pole – point out the pole out to learners, noting that there is a right angle at ground level. If the right angle had not been filled in, learners must know that a pole will stand at a aright angle to the ground and should mark the angle as a right angle.

'Same horizontal plane' means they are on the same level - which is the ground or the floor.

Read the information about the wire cables and point them out. Learners may want to label the cables.

Discuss the angle of depression – imagine a cat is sitting at the top of the pole looking down.

Ensure all measurements mentioned in the question are marked on the diagram:

a) Angle of depression is formed from the horizontal which will in turn be parallel to the ground. Therefore, alternate angles are equal. Tell learners to fill the answer in on the diagram.
b) Although QR lies in two right-angled triangles, only one of these triangles ($\Delta\mathrm{PQR}$) has sufficient information to find the length of QR. According to the angle named ($\theta/47^{o}$), we are looking for the opposite and are given the hypotenuse. This requires the use of the trig ratio, sine. Tell learners to fill the answer in on the diagram.
c) Ŝ lies in the right-angled triangle QRS. According to the angle required, we have the opposite and adjacent. This requires the use of the trig ratio, tangent.
   Tell learners to fill the answer in on the diagram.
d) To calculate this distance, we require the length of PQ. According to the angle named ($\theta/47^{o}$), we are looking for the adjacent and are given the hypotenuse. This requires the use of the trig ratio, cosine. Point out that learners could also use sine by using the 3rd angle in the triangle OR they could use the length of QR but using an answer that has been calculated during the question when a measurement given can just as easily be used is not ideal in case an error was made in the calculation.
   The theorem of Pythagoras could also have been used.

Solutions:

a) $\theta = 47^o$

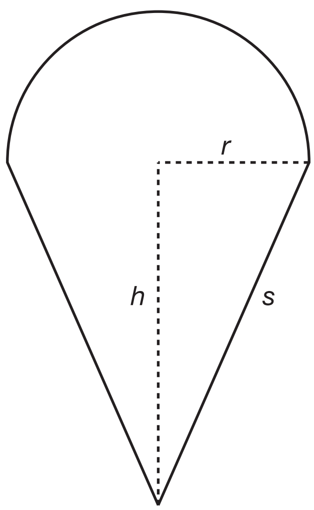
b)
$$\sin \mathrm{P} = \frac{\mathrm{RQ}}{\mathrm{RP}}$$
$$\sin 47^o = \frac{\mathrm{RQ}}{21}$$
$$\mathrm{RQ} = 21 \sin 47^o$$
$$\mathrm{RQ} = 15{,}36\mathrm{m}$$

c)
$$\tan\ \mathrm{S} = \frac{\mathrm{RQ}}{\mathrm{QS}}$$
$$\tan\ \mathrm{S} = \frac{15{,}36}{17}$$
$$\therefore \hat{\mathrm{S}} = 42{,}1^o$$

d)
$$\cos 47^o = \frac{\mathrm{PQ}}{21}$$
$$21 \cos 47^o = \mathrm{PQ}$$
$$\therefore PQ = 14{,}32$$
$$\therefore PS = 14{,}32 + 17$$
$$= 31{,}32\mathrm{m}$$

EUCLIDEAN GEOMETRY AND MEASUREMENT

The diagram below shows the cross-section of a solid made up of a right circular cone with radius r and slant height s. The perpendicular height of the cone, h is 6,5cm and the volume of the cone is 83,38cm^3.



Formulae:

Surface area of sphere $= 4\pi r^2$

Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of cone $= \pi r^2 + \pi rs$

Volume of cone $= \frac{1}{3}\pi r^2 h$

Calculate, correct to two decimal places:

a) The radius, r of the cone

b) The slant height, s, of the cone

c) The surface area of the solid.

Teaching notes:

Tell learners to check that they understand all the information in the question and whether each piece of information is represented on the diagram. If not, they should add to the diagram.

a) Ask: *What will you need to do to find the radius?*
(Volume and height are provided, so you can use the formula for volume and work in reverse to find the radius).

b) Learners should recognise that this is a theorem of Pythagoras question.

c) Ask: *How can you find the volume of the complex shape?*
(Volume of the cone added to volume of the hemisphere – half a sphere).
Ask: *Do you have all the measurements available for this?*
(Yes –radius and the slant height).
Confirm that learners can see that they won't need the base circle of the cone's formula as it is 'inside' the solid.

Solutions:

a) Volume of cone $= \frac{1}{3}\pi r^2 h$

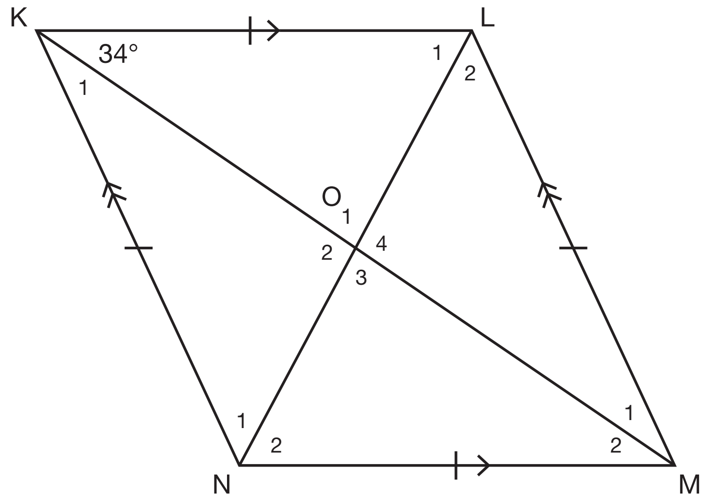
$$83,38 = \frac{1}{3}\pi r^2 (6,5)$$
$$250,14 = 6,5\pi r^2$$
$$\frac{250,14}{6,5\pi} = r^2$$
$$r = 3,5cm$$

b)
$$r^2 + h^2 = s^2$$
$$(3.5)^2 + (6,5)^2 = s^2$$
$$54,5 = s^2$$
$$s = 7,38cm$$

c) Surface area $=$ cone (without base) + hemisphere

$$= \pi rs + \frac{1}{2}(4\pi r^2)$$
$$= \pi(3,5)(7,38) + 2\pi(3,5)^2$$
$$= 158,12\text{cm}^2$$

KLMN is a rhombus with diagonals intersecting at O. $L\hat{K}M = 34^o$



a) Write down the size of $\hat{O}_1$.
b) Calculate the size of $\hat{L}_1$.
c) Calculate the size of $K\hat{N}M$.

Teaching notes:
Tell learners that if they know their properties of quadrilaterals (and a rhombus in in this case), as well as their Grade 8 Geometry, this should be an easy question. Ask learners what they know about a rhombus. As they give you information, fill it in on the diagram and ask them to do the same. Note that some information is already marked on the diagram, so is not mentioned here.

- Opposite angles equal
- Diagonals bisect angles
- Diagonals bisect each other at 90°

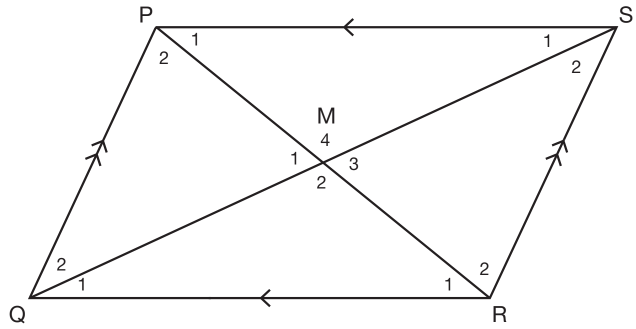
Solutions:

a) $\hat{O}_1 = 90^o$ (diagonals bisect at 90°)

b) $\hat{L}_1 + 34^o + 90^o = 180^o$ (<'s of Δ))
$\hat{L}_1 = 56^o$

c) $\hat{L}_1 = \hat{L}_2 = 56°$ (diagonals bisect angles)
$\therefore K\hat{N}M = 112^o$ (oppos <'s equal)

Given parallelogram PQRS with diagonals PR and QS intersecting at M.



Prove that the diagonals bisect each other.

Teaching notes:

Ask: *For the diagonals to bisect, what needs to be equal?* (QM & MS and PM & MR)

Ask: How can we prove that these lines are equal in length?

(Use congruency in any opposite pair of triangles).

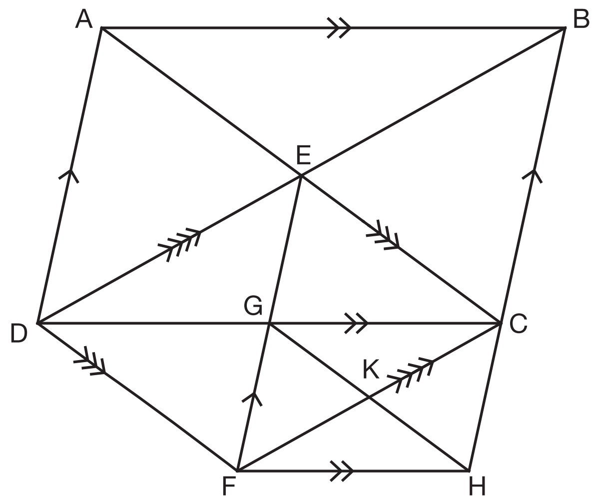
Point out how important it is to know geometry from previous years – both this question and the previous question required this knowledge.

Solution:

In ΔPQM and ΔSRM

$PQ = SR$	(opp sides of parm)
$\hat{P}_2 = \hat{R}_2$	($PQ \parallel SR$; alt <'s equal)
$\hat{Q}_2 = \hat{S}_2$	($PQ \parallel SR$; alt <'s equal)
$\therefore \Delta PQM \equiv \Delta RSM$	(AAS)
$\therefore QM = SM$ and $PM = RM$	($\Delta PQM \equiv \Delta RSM$)

In the diagram, ABCD is a parallelogram with diagonals intersecting at E. The diagonals of parallelogram DECF intersect at G. The diagonals of parallelogram FGCH intersect at K.



Prove that DB $=$ 4KC

Teaching notes:

This diagram can be confusing for learners. Learners often find it useful to use colours.

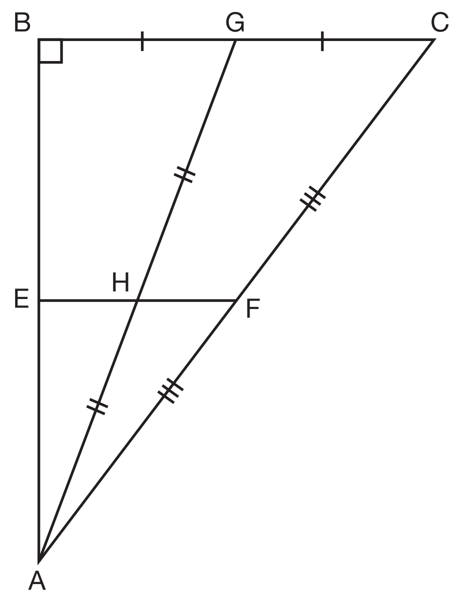
Tell learners to note that three parallelograms, as well as their diagonals and where they intersect, are mentioned. This should be a clue as to what knowledge may be useful to prove this rider.

Work through this question with learners. Show how the first parallelogram is used to make a statement which leads into the second parallelogram and, finally, into the third parallelogram to involve 'KC'.

Solutions:

	$DB = 2DE$	(diagonals bisect each other)
	$DE = FC$	(oppos sides of parm)
But	$FC = 2KC$	(diagonals bisect each other)
	$DE = 2KC$	$(DE = FC)$
	$DB = 2(2KC)$	$(DB = 2DE)$
	$\therefore DB = KC$	

ΔABC is a right-angled triangle at B. F and G are the midpoints of AC and BC respectively. H is the midpoint of AG. E lies on AB such that FHE is a straight line.



a) Prove that E is the midpoint of AB.

b) If $EH = 3{,}5$cm and the area of $\Delta AEH = 9{,}5\text{cm}^2$, calculate the length of AB.

c) Hence, calculate the area of ΔABC.

Teaching notes:

Ask: *What theorem do you recognise from this diagram and information?*

(The midpoint theorem)

Before looking at the questions, discuss which lines are parallel and which lines are half (or double) the length of another line. Encourage learners to do this in an exam situation – to study the information and diagram and mark what they know before approaching the actual question(s).

a) Ask: *What do we really need to prove in order to prove that E is the midpoint of AB?* (That AE = BE). This can easily be done using the midpoint theorem.

b) Ask: *How do we find the area of a triangle?* (Area $\Delta = \frac{1}{2}b. \perp ht$).
Point out that we are given area and the base. It is therefore possible to find the height (AE). This will lead to finding AB. Remind learners that the base and height of a right-angled triangle are always the sides that form the right angle.

c) Ask: *What measurements are required to find the area of* ΔABC*?*
(AB and BC as they are the base and height).
Do we have those measurements? (Yes, we have AB from the previous question and we can find BC using the midpoint theorem).

Solutions:

a) In ΔACG:
FH ∥ CG (midpoint theorem)
∴ EF ∥ BC (same lines)
In ΔAGB:
AE = EB (line through midpt ∥ to 2nd side)
∴ E is the midpoint of AB.

b) Area $\Delta\text{AEH} = \frac{1}{2}b \times \perp ht$

$$9{,}5\text{cm}^2 = \frac{1}{2}(3{,}5\text{cm})(\text{AE})$$
$$19\text{cm}^2 = (3{,}5\text{cm})(\text{AE})$$
$$\frac{19\text{cm}^2}{3{,}5cm} = \text{AE}$$

∴ AB = 2AE (AB = AE; proved above)
∴ AB = 10,86cm

c) BG = 7cm (midpoint theorem)
∴BC = 14cm
Area $\Delta\text{ABC} = \frac{1}{2}b \times \perp ht$
$$= \frac{1}{2}(14) \times (10{,}86)$$
$$= 76\text{cm}^2$$

8. When the past paper has been completed ask learners if they have any questions.

9. Say: *Next week you will be given an opportunity to work through past papers on your own.*

REVISION – WEEKS 3 & 4

POLICY AND OUTCOMES

A

CAPS Page Number	29
Lesson Objectives By the end of the lesson, learners will have: ● worked on your own to complete a Paper 1 past paper ● worked on your own to complete a Paper 2 past paper ● made 'cheat sheets' covering all topics.	

CLASSROOM MANAGEMENT

B

1. Make sure that you are ready and prepared.
2. Advance preparation:
   - work through the past examination papers that the learners will be doing on their own. This is essential to assist them quickly and smoothly when learners need help.
   - work through the instructions on how to make a cheat sheet to assist them in their studying.
3. The examinations and memoranda are available in the Resource Pack (Resources, 6, 7, 8 and 9) for photocopying if possible.
4. Write work on the chalkboard before the learners arrive to ensure no time is wasted.

CONCEPTUAL DEVELOPMENT

C

INTRODUCTION

1. Learners have now done revision with your help. It is time for them to try past papers on their own.
2. As learners work through the past papers they should make 'cheat sheets'. Explain how to go about this to them before they start on the past papers.

DIRECT INSTRUCTION

1. Start the lesson by saying: *Now that we have spent two weeks doing revision together, it is time for you to work on your own. I will be available to assist you but mostly you need to work alone*.

2. Tell learners that while they are working through the papers, they should make cheat sheets. Make it clear that cheat sheets are a study aid and not notes that can be used for cheating!

3. Go through the following instructions with learners.

   A cheat sheet is a document (generally only one-page front and back) that contains all the key information that is likely to be in an assessment. Even though you can't use the cheat sheet in the examination, the preparation of a cheat sheet is a great way to prepare for the exam.

   Guidelines for preparing a cheat sheet:

   1. Develop the cheat sheet gradually by adding new items as you work through the past papers.
   2. Write out the cheat sheet by hand. You can get more on the document that way. At the end of each past paper, re-do the cheat sheets for each topic and put them in a safe place with your summary notes as well as any other study notes.
   3. Include the following items on your cheat sheet.
      - Formulas
      - Example problems worked out
      - Steps used in the problem listed in order
      - Reminders of things to look out for in doing a problem
      - Rules used to solve problems
      - Definitions
      - Types of problems that you know will be in an examination.
   4. If you recall problems you struggled with in the past, be sure to include information on these.
   5. Use your past papers as a guideline to prepare your cheat sheets. Past papers are all set along the same lines. Now that this is your 2nd past paper for each exam (Paper 1 and Paper 2), you will start noticing what is often assessed.
   6. While you are working through the past paper, refer to your exercise book, text book and summary notes for cheat sheet information.

7. Find a method to compartmentalise items. For example, highlight what you need to memorise in one colour and tips in another colour. Use bullet points and different-sized headings. Find a layout that suits your study method.
8. Choose whether your cheat sheet is a summarised list (like the summary notes you have already received) or a mind map.
9. Review your cheat sheet and summary notes for at least one hour every day for a week ahead of the exam. This continual review will help you remember the concepts.
10. Use your cheat sheet as the primary study resource for the final. If you have kept these up-to-date, you should be able to reduce your preparation time for finals.

4. Hand out both the past papers (Paper 1 and Paper 2 – resources 6 and 7). Allow learners to choose where they start. Point out, however, that by the end of the 7 or 8 days they must have completed both papers as well as their cheat sheets. You will need a few days for the past papers to be marked and corrected.

5. If photocopying is readily available, you can photocopy a few memoranda (Resources 7 and 8) and, with 2 or 3 days to go, allow learners to sit in groups to mark and discuss the solutions.If photocopying is not an option, the learners could be given the link (providing some of them have data) and marking could still take place in groups.

6. If neither of these options are possible, you can call out the final answers of each question then ask learners which they would like you to do in full with them.

7. Verbalising problems and sharing their own knowledge with others can be a very effective learning tool.

8. The correcting of the papers is an essential part of the exercise. There is little value in doing questions and not knowing if they have been done correctly or not. When marking does take place, tell learners that they must do the full corrections for any question that was incorrect.

9. Learners should add notes to their cheat sheets relating to any mistakes they made.

10. We wish you and your learners well.